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GEOMETRICAL OPTICS.

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GEOMETRICAL OPTICS

ADAPTED TO THE USE OF THE HIGHER
CLASSES IN SCHOOLS, &c.

BY

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BIBLIOTHECA BODLEIANA

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PREFACE.

THIS is, I imagine, the first time that any attempt has been made to adapt the subject of geometrical optics to the reading of the higher classes in our good schools. That this should be so is the more a matter for remark, since the subject would appear to be peculiarly fitted for such an adaptation. The great simplicity of its primary laws, in the first place, and the very small amount of analysis which, generally speaking, they involve, render the commencement not so unattractive as that of many subjects which are usually taken as the beginning of the second course of mathematical reading: whilst the new ideas and trains of thought which are introduced at every stage appear to me to be at once interesting and valuable. The conception of a virtual image, to take an early instance, is probably an entirely new one to the reader's mind. So also is the idea of a caustic curve, and the subject abounds with similar new considerations.

But the chief advantage that this subject possesses appears to me to be the middle position which it holds between the purely theoretical and the purely experimental. It contains sufficient of physical interest to give reality, and the easy and certain experiments by which it can be illustrated are convincing evidences of the correctness of the results, whilst the analysis which is requisite to obtain these results is sufficient to afford the reader, who up till now has been studying Algebra and Euclid, a proof that the said Algebra and Euclid have really some distinct use in explaining the phænomena of common life.

When the idea of writing such a book was first suggested to me by my old master, Professor Drew of King's College, I was afraid that there was hardly room, under the present system, for the subject in the course of a boy's school-work: I therefore, after making some progress in it, wrote to obtain the opinion of the head mathematical masters of two or three of our best schools; and I cannot sufficiently acknowledge the courtesy with which these gentlemen gave me the information I required. While acknowledging that the doubt which I had expressed existed in their own minds, they urged me strongly to continue my work, as they fully agreed in recognizing the great use of introducing a subject into school-work which should combine new ideas with practice in former knowledge.

The familiar use of the subject of Geometrical Optics by all who have received any but a very slight mathematical training precludes the possibility of originality in the facts themselves, and admits of but small originality in their treatment. At the same time it is hoped that it will be found that this book is not merely a rescript of any existing work.

It might be thought that it would be an easy task to adapt a subject of simple principles to the comprehension of boys possessing mathematical knowledge of the standard usually found in the higher classes of our schools. I imagined so myself, and during the course of my working I have been gradually learning my error. I can only wish that what I consider to be an object of the highest importance had been approached by some one more fitted both by mathematical knowledge and experience than by one whose sole qualifications are considerable fondness of the subject and a great desire to afford to beginners some glimpse of one of the steps to science to which their daily mathematics are leading. I have endeavoured as much as possible to avoid the example of those popular lecturers who explain difficulties by ignoring them. But as the nature of my design necessitated brevity, I have omitted entirely one or two portions of the subject which I considered unnecessary to a clear understanding of the rest, and which appear to me better learnt at a more advanced stage,—such are the subjects of *Illumination of Surfaces* and *Achromatism.*

I need not say that I shall feel most thankful for notices of corrections that are needed, and for any suggestions as to the improvement of the book.

O. AIRY.

WELLINGTON COLLEGE,
April 2, 1870.

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GEOMETRICAL OPTICS.

CHAPTER I.

INTRODUCTION.

THERE are certain ideas connected with light and vision which are familiar to us all from the moment that our powers of observation exist. We know, for instance, that some bodies send out light of their own, such as the sun, or a gas-flame, or a red-hot coal; whereas others send out only light which they have received from something else, as, for instance, the bright spots on the polished back of a chair which come from the light of the gas-flame, or the fire, or the moon, who sends us the light which she gets from the sun. Then again, we cannot but feel sure that light comes from the object to our eye in straight lines. As an illustration of this there is the fact that if a small round hole be made in a sheet of paper, and a steady candle-flame held before it, the light which passes through the hole will make an image of the flame upside down on another sheet of paper held parallel to the first. But this is only true so long as we suppose the light to be passing through the same "medium," as it is called, that is, for instance, so long as it is passing through the same kind of air. If in its course it come to a surface of water, the same thing would happen to it as happens to a ball passing through the air which is suddenly struck by another ball thrown from the side; *i.e.* its course is instantly changed from the straight line in which it *was* travelling to another straight line.

2. Before going any further we will give the definition of some terms which we shall often have to use :—

(i) Any point of a luminous body is sending out light in every direction in straight lines. It is therefore called a "*centre of light*," or a "*luminous point*."

(ii) And any one of such straight lines is called a "*Ray*."

(iii) Any number of consecutive Rays form a "*pencil*." The most convenient form in which to imagine a pencil is a solid cone of rays, of which the centre of light is the vertex.

(iv) The middle ray, or axis of the cone of rays, is called the "*Axis*" of the pencil*.

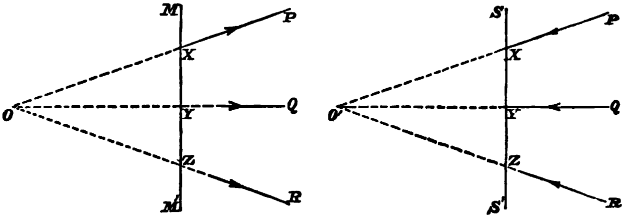
(v) When the rays forming a pencil of light come *from* a point, or *as if from a point*, the pencil is called a "*divergent*" pencil. When they come *to* a point, or *as if to* a point, it is called "*convergent*."

---

* The ancients used to assume that the rays of a pencil were not close together, but separated like the fingers of the hand spread. By this they accounted for the fact that occasionally, in looking for a small object, like a needle, we cannot see it when it is under our nose.

Thus if *MM'* were a mirror, it is possible to see that rays might come off that mirror as if from *O*; or to fall on the screen *SS'* as if they were going to *O'*. Such pencils are respectively divergent and convergent, as much as if the rays really started from *O* or met at *O'*.

Fig. 1.



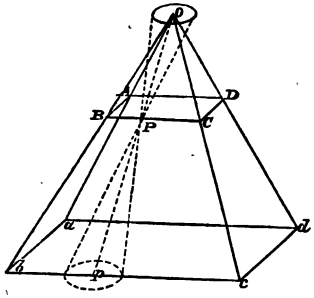
We shall often have occasion to talk about pencils of light falling upon plane surfaces and others: in all such cases, unless expressly mentioned, we shall suppose that the axis of the pencil falls perpendicularly on the surface.

(vi) Any substance through which light passes, such as air, water, &c., is called a "*medium*," and it is called "*uniform*," when its density, upon which depends the amount of bending that a ray suffers (Art. 2), is the same at every point. All media at present are to be considered uniform.

3. The following example may serve to familiarize the reader with the idea of a ray and a pencil of light. "Light emanating from a luminous circular disc placed in the ceiling of a room, passes through a rectangular aperture in the floor. Ascertain the form of the luminous patch on the floor of the room below."

(i) Let O be the centre of the disc; and let a straight line of light from O travel round the hole $ABCD$: let $abcd$ be the limit of light traced out by it upon the floor below.

Fig. 2.



(ii) Take any point P on the boundary of the hole, and ~~think~~ of all the rays which come from the disc and pass through P. They will make a circular patch at p on the boundary found in (i).

(iii) Let P travel all round $ABCD$: then the circular patch travels all round $abcd$, and radius of circular patch : radius of disc $= Pp : PO$.

If the hole be large compared with the disc, the travelling circle will be small, and the patch will therefore be similar in shape to that of the aperture, but with rounded corners.

If the disc be large compared with the hole, the general appearance will be a circle, whatever the shape of the aperture.

This example has an historical interest of its own. The first explanation was given by Maurolicus of Messina, in 1575, in words tantamount to our explanation. The following is translated from Monteucla's *History of Mathematics* (Part iii. Bk. v. 2):

"Why, asked Aristotle, does a ray of the sun, after passing through a hole of any shape, triangular for instance, or even a mere straight slit, if it is intercepted at a certain distance, always form a circle? and, still more strange, why, when the sun is partly eclipsed, does this ray in passing through the same hole as before, form an image precisely similar to the part of the sun's disc which is not yet hidden? This problem, which had been up to that time the despair of philosophers, had compelled them to say with Aristotle, that apparently the light took a certain curvature, or a similarity to the luminous body, which it regained immediately on freeing itself from the obstacle which had forced it from its shape......To explain this phenomenon, we will first notice, with Maurolicus, that each point of the aperture is the vertex of two opposite cones, one of which has the sun for its base, and the other, if cut by a plane perpendicular to its axis, will produce a luminous circle, the size of which varies as the plane of intersection is removed further from the opening. Thus there are depicted on the plane as many equal circles of light as there are points in the opening. And thus, if we draw on this plane a figure equal and similarly placed to that of the aperture, and at each of its points or only at those on its boundary describe a multitude of circles, the figure which they will form will be precisely that of the image of the sun, received at a distance proportional to the size of these circles. But in proportion as we increase the size of our circles, we shall see that the figure which results from them will approach nearer to a plain circle, and this is easy of demonstration........... But why does the sun, in a partial eclipse, form in a dark room the figure of a crescent, whatever is the shape of the opening? The explanation is the same as that just given. If we have a figure of any shape on a plane, and if from each of the points of its boundary we describe a succession of other figures similar to each other, and similarly placed, the resulting figure will approach nearer to that of those which are the greatest. If these are triangles, the whole figure will be a triangle; if a crescent, the figure formed will resemble a crescent."

CHAPTER II.

PLANE REFLEXION.

4. WHEN rays of light fall upon any surface, they are either sent back from the surface, or else they enter the surface: in the first case they are said to be "*reflected*" from the surface, in the latter case they are said to be "*refracted*" into the surface. At present we will only consider the laws and some of the phenomena of reflexion.

Some surfaces, such as polished metal, do not refract at all: others, like water, partly reflect and partly refract, the amount reflected bearing a proportion to the amount refracted, depending on the angle at which the rays meet the surface. Thus, if a number of rays fall perpendicularly on to the water they are nearly all refracted: if they fall at an angle to the water, some of them are reflected, and the more their direction slopes to the water the more rays are reflected, until at last none are refracted at all.

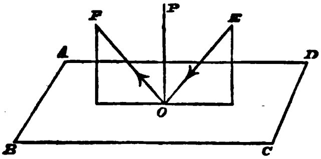
5. Suppose a single ray falls upon any reflecting surface; then

(i) The incident and reflected rays lie in one and the same plane with the normal to the surface at the point of incidence, and on opposite sides of it.

(ii) The angles which the incident and reflected rays make with the normal are equal to one another.

Thus, if *OP* be a normal to *ABCD* at *O*, and if a ray of light *EO* fall upon the surface at *O*, and is reflected in the direction *OF*, then *EO*, *OP*, and *OF* are all in the same plane, and the angle *FOP* = angle *EOP*.

Fig. 3.



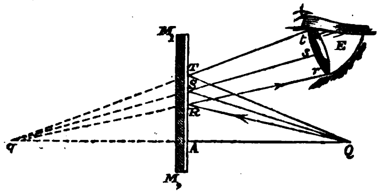
Thus the action of a reflecting surface upon a ray of light is precisely the same as that of a wall upon a very elastic ball thrown obliquely against it. In playing racquets, for instance, one naturally, without thinking of it, places oneself so as to receive the ball after coming off the wall at about the same angle as it went on; and the more elastic the ball is, the more nearly is this true; and if the ball is very elastic indeed, it is very nearly true; and when instead of a ball we think of a ray of light, which is quite elastic, this is absolutely true.

From these two properties, which were known by the ancients as well as by ourselves, all the phenomena of reflexion can be explained. We will consider a few of the simplest—at present supposing the reflecting surface to be quite plane.

6. The simplest and at the same time the most common instance of plane reflexion is that of a looking-glass; in which we appear to see the image of any object, placed in front of the mirror, just as far behind the mirror as the object itself is in front.

Let M_1M_2 be a mirror, edgeways to us. Let Q be a point of an object in front of the mirror: Q is sending out rays in all directions. E is an eye looking into the mirror. Draw QA perpendicular to the mirror. Let any ray QR strike the mirror at R: it will be reflected back in the line Rr, where $\angle M_1Rr = M_2RQ$. Produce Rr backwards through the mirror, to meet QA produced in q. Then the angle $M_1Rr =$ angle M_2RQ and also $=$ angle qRA; $\therefore qRA = M_2RQ$, and the right angle $qAR = RAQ$, and the side RA is common. Therefore, by Euclid I. 26, the other sides, &c. of the triangles are equal; $\therefore qA = QA$.

Fig. 4.



So if we take any other rays QS, QT... they will be reflected as Ss, Tt, and these lines when produced backwards will all pass through q.

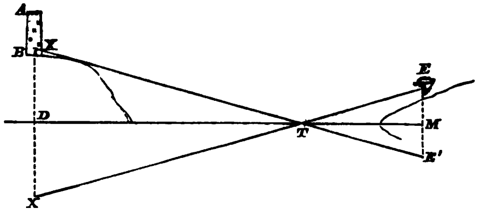
Now another way of saying this is to say that the eye at E will be affected by these rays in just the same way as if they all started from q. That is the eye will see an image of Q at q, just as far behind the mirror as Q is in front, since $Aq = AQ$.

In this we have taken only a small piece of the divergent pencil from Q, of which QA is the axis. After striking the mirror it becomes a divergent pencil from q, according to our definition of a divergent pencil in Art. 3 (v), for the rays meet the eye as if they came from one point.

An image through which the rays do not really pass, as q, is called "*virtual.*" Those through which the rays really pass are called "*real.*" Thus all the images formed by plane reflexion are virtual. It should be noticed that if we are considering the rays after reflexion, we may practically consider Q and the mirror to be entirely removed, and the point q to be a real point sending out luminous rays.

Suppose that AB was a tower on some high ground on one side of a piece of water, and that E was an eye on the other side looking into the water: will the eye see the whole of the tower by reflexion?

Fig. 5.



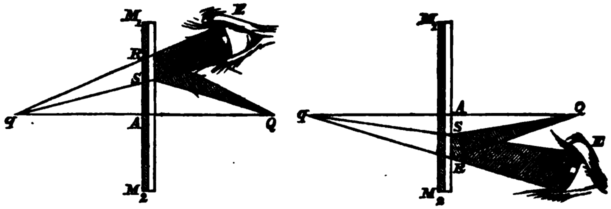
The eye at E will clearly see just as much as an eye at E' where $ME' = ME$.

7. If it is wished to trace the course of the rays which come from the object to the eye, before and after reflexion, we can at once obtain the construction from Fig. 4. It is this. "Join the image with the eye, by straight lines, and shade between these lines from the mirror up to the eye. From the points where these lines cut the mirror, draw straight lines to the original point, and shade. The shaded part will represent the required rays."

Thus, using the same letters as before, join q the image with the eye E: and shade from R and S to the eye. Join R and S to Q and shade. The shaded part represents the rays which fall from Q on the mirror, and thence come to the eye.

Whenever, in future investigations, we require to trace the course of a pencil of rays from an object to the eye, after any number of reflexions, we shall make use of an extension of this rule—viz.:

Fig. 6.



(1) Join the eye to last image, and shade to the eye from the points where these lines cut the last surface.

(2) Join these points to last image but one, and shade from them up to last surface but one.

(3) Join the points last found to last image but two, and shade from them up to last surface but two, and proceed in this manner, until we come to the first surface, when we join the points found in it to the object, and shade as before.

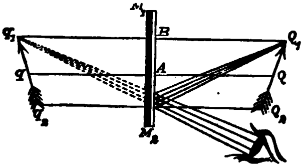
We thus, in fact, trace the rays back from the eye to the object.

8. The result of Article 6 applies, of course, to any point of an object. Thus every point of an object has an image of itself formed just as far behind the mirror as the point itself is in front of the mirror.

Thus, if Q_1QQ_2 be any object placed in any position in front of the mirror M_1M_2, it will have an image of itself at q_1qq_2 where

$$Aq = AQ,$$
$$Bq_1 = BQ_1,$$
$$Cq_2 = CQ_2.$$

Fig. 7.

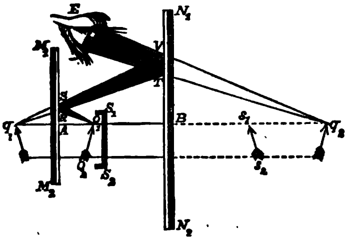


The images q_1, q, and q_2 all being virtual.

9. Let M_1M_2 be a mirror, Q_1Q_2 any object in front of it, the image of which is at q_1.

Let N_1N_2 be another mirror, exactly facing the former, but let there be a screen S_1S_2 so placed that no rays can come from Q_1Q_2 on to N_1N_2; *i.e.* so that no image of Q_1Q_2 can be formed by direct reflexion at N_1N_2.

Fig. 8.



Consider any point Q_1. Rays of light Q_1R, Q_1S, come from Q_1 on to the mirror M_1M_2 and are reflected from it as if they came from q_1, where $Aq_1 = AQ_1$. They then fall on N_1N_2 as RT, SV. Now since they may be considered as rays diverging *from the point* q_1, SV and RT will of course form an image of q_1 at q_2, where $Bq_2 = Bq_1$, which will be seen by an eye looking into N_1N_2.

Thus the rays which reach the eye come as if from q_2, *i.e.* the eye E sees an image q_2 of Q_1. And similarly for all the other points of Q_1Q_2.

The shaded part represents the rays traced from Q_1 to the eye E, by means of the rule in Art. 8.

We can easily find the distance of q_2 from either mirror.

For $Aq_1 = AQ_1$; $\therefore q_1B = 2AQ_1 + Q_1B$;

$$\begin{aligned} \therefore \text{ also } q_2B &= 2AQ_1 + Q_1B \\ &= 2AQ_1 + AB - AQ_1 \\ &= AQ_1 + AB; \\ \therefore q_2A &= AQ_1 + 2\,.\,AB. \end{aligned}$$

10. If the rays were again allowed to fall on the mirror M_1M_2 before coming to an eye, the eye would see an image q_3 behind the mirror M_1M_2, such that

$$Aq_3 = Aq_2 = AQ_1 + 2AB.$$

If again they fell upon N_1N_2 before coming to the eye, a fourth image q_4 would be formed as far behind N_1N_2 as q_3 is in front; whence $q_4A = AQ_1 + 4AB$, and so on.

In this investigation we have carefully considered only a pencil of rays which fall *first* on the mirror M_1M_2, in order to save confusion: but it is easy to see that in precisely the same manner there will be a series of images formed by a pencil of rays which fall first on N_1N_2, of which s_1s_2 would be the first.

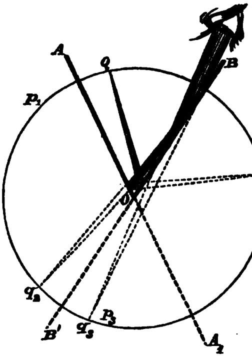
It is perhaps necessary to remark that difficulty may be felt at the phrase "the image *formed* by reflexion." It will be felt that this is a wrong word to use, since the rays do not *actually* pass through q_1, q_2, &c. Of course there is no image formed—but if an eye is placed so as to catch the rays which come as if from one of the points q_1, q_2, &c. an image is *practically* formed there.

Explanation of the Kaleidoscope.

11. (1) An object is placed between two mirrors inclined to one another at any angle. To find the number and positions of the images.

Let OA and OB be the mirrors. Let Q be the object, placed between them. Let us consider only light falling first on OB.

Fig. 9.



Then, of course, Q has an image q_1, where q_1 is as far behind OB as Q is in front, *i.e.* after being reflected at OB the rays will appear to diverge from q_1.................... (1).

Next, these rays will now fall on OA, and, since they practically come from q_1, they will, after reflexion at AO, appear to diverge from q_2, where q_2 is as far behind OA as q_1 is in front of it.. (2).

Lastly, these rays will now fall on OB, and, since they practically come from q_2, they will, after reflexion at OB, appear to diverge from q_3, where q_3 is as far behind OB as q_2 is in front of it.

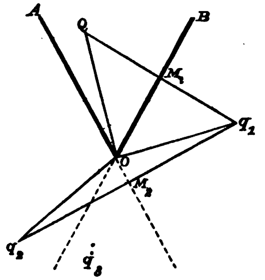
We say *lastly*, because it is clear that no rays which come off OB as if from q_3 can possibly fall on OA, and there will therefore be no more images.

The figure shews the course of a small pencil which comes to the eye after suffering the greatest number of reflexions possible. On entering the eye the rays seem to diverge from q_3, *i.e.* the eye sees an image there.

The eye will, of course, see *all* the images, although we have traced only the course of the pencil by which it sees the last one. Rays, for instance, will fall from Q on OB and be reflected directly to the eye, which will thus see q_1. Similarly rays will come to the eye after two reflexions, and the eye will therefore see q_2. The student should draw figures for himself of these pencils, using the rules given in Article 8.

(2) We will now shew that all these images lie upon a circle whose centre is O and radius OQ.

Fig. 10.



For $QM_1 = q_1M_1$ and M_1O is common; ∴ QM_1, $M_1O = q_1M_1$, M_1O, each to each, and the angle QM_1O = angle q_1M_1O; ∴ the base QO = the base q_1O.

In just the same way, since $q_1M_2 = q_2M_2$ and OM_2 is common, and the angle q_1M_2O = angle q_2M_2O, therefore $q_1O = q_2O$.

Thus $q_3O = q_2O = q_1O$, that is the points q_1 q_2 q_3 all lie on a circle, centre O, and radius OQ.

It must be observed that hitherto we have only considered rays which fall upon OB first. If we now consider those which fall first on OA we shall obtain three more images P_1 P_2 P_3. It can be shewn that if the mirrors are inclined to one another at any submultiple of two right angles, 30°, 60°, for example, the last images of the two systems of rays will coincide. Thus if our mirrors are inclined at 60°, the eye will see five images and not six (see figure in next article).

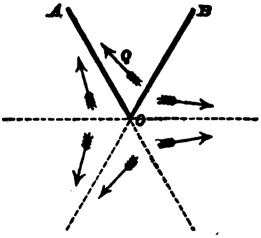
12. The positions of these images can be formed generally, supposing that we know the angles QOA and QOB.

For
$$QOq_1 = 2 \,.\, QOB \quad\text{..................}\quad \text{(i)};$$
$$\therefore q_1OA = 2 \,.\, QOB + QOA;$$
$$\therefore q_2OA = 2 \,.\, QOB + QOA;$$
$$\therefore QOq_2 = 2 \,.\, QOB + 2 \,.\, QOA \quad\text{......}\quad \text{(ii)},$$
similarly
$$QOq_3 = 2 \,.\, QOB + 2 \,.\, QOA + 2 \,.\, QOB \quad\text{......}\quad \text{(iii)},$$
$$QOq_4 = 2 \,.\, QOB + 2 \,.\, QOA + 2 \,.\, QOB + 2 \,.\, QOA,$$
so that the angular distances of $q_1\, q_2\, q_3\, q_4 \ldots$ from OQ are found each from the one before it, by adding alternately $2 \,.\, QOA$ and $2 \,.\, QOB$.

The figures which we have drawn are of course sections of the mirrors perpendicular to the line in which they meet. Although these images exist for an eye in the same plane as the section, yet in order that they might be seen by an eye out of the plane, the mirrors would have to be prolonged, so that rays, after striking them, might meet the eye.

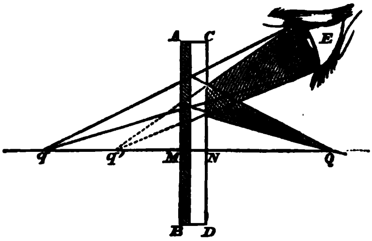
So that a virtual image may be described as an image that does not exist until there is an eye to receive the rays. The eye calls the image, though not the rays, into existence.

Fig. 11.



13. We can explain the reason why, when looking into a glass mirror, we very often see a double image of an object, one image being very faint.

Fig. 12.



Let *ABCD* be the mirror. Let *QNM* be perpendicular to the mirror. *AB* is the silvered back.

The strong image is formed by the light which, passing into the glass, is reflected from AB. But a very little of this light never gets to AB through CD at all, but is reflected from CD. Thus we obtain another very faint image q', where $q'q = 2MN$.

For
$$Mq = MQ = MN + NQ,$$
$$Nq' \quad = NQ\,;\ \therefore\ Mq' = NQ - MN\,;$$
$$\therefore\ qq', \text{ which} = Mq' - Nq' = 2MN,$$
$$\textit{i.e. } qq' = \text{twice thickness of the plate.}$$

But at present in talking of glass mirrors we shall always neglect the thickness of the plate. The shaded portion of this figure is intended to represent the rays forming the faint image.

Note. We have here, for convenience of explanation, neglected the refraction which the light suffers on entering, and emerging from, the glass.

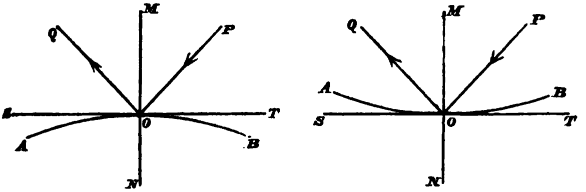
CHAPTER III.

REFLEXION AT A CURVED SURFACE.

14. We have hitherto thought only of the images formed by rays which fall upon and are reflected from plane mirrors, and have seen that in all such cases we shall have an image of any object just as far behind the mirror as the object itself is in front; and the only rule which we used to find this was that after a ray of light struck the mirror it came off it again, making the same angle with the mirror or with a line perpendicular to the wall as it did on striking it.

Now precisely the same thing happens to a ray when it strikes a curved surface; but the result is not the same, *i.e.* the image is not seen just as far behind as the object is in front.

Fig. 13.



Thus let *AOB* be the section of any curved surface, made by the plane of the paper, and let a ray from *P* strike the surface at *O*: draw a plane of which the section is

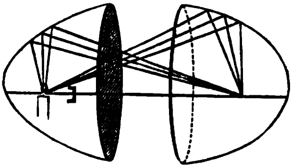
ST touching the curved surface in O, and let MON be a line perpendicular to ST: then if OQ be the ray after reflection $Q\hat{O}M = P\hat{O}M$; and the direction of OQ is therefore just the same as if the ray had struck the plane ST instead of the curved surface.

(It will be very useful sometimes when we have a ray striking a curved surface, to treat it as if it struck the plane surface which touches the curved surface at the point.)

15. (i) The simplest case of a curved surface is a *sphere.* Suppose a light placed in the centre of a sphere. A line from the centre to any point on the circumference is perpendicular to, that is, makes equal angles with, the tangent at that point. But the incident and reflected rays also made equal angles with the tangent, whence it follows that the reflected ray returns along the incident ray, *i.e.* all the reflected rays pass through the centre.

(ii) Take an *elliptical* surface with a light in one focus, and suppose a ray to fall on the reflecting surface. Now we know by Optics that the incident and reflected rays make equal angles with the tangent. Also we know by Geometry that the lines from the foci make equal angles with the tangent; whence

Fig. 14.



it follows that a ray incident from one focus is reflected to the other—and similarly for all the rays.

An interesting experimental proof is the following. If a candle flame be placed in one focus and a piece of phosphorus in the other, the concentrated reflected rays of heat, which follow the same law as light as regards reflexion, will ignite the phosphorus, although the direct light of the candle be kept away by a screen.

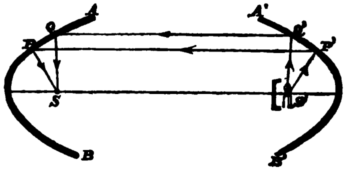
(iii) Consider a *parabolic* mirror with a light in the focus. By precisely similar reasoning it may be seen that all the reflected rays will go off in a beam parallel to the axis, the breadth of the beam of course depending on the size of the parabola.

Upon this principle the construction of one kind of lighthouse is based. A mirror very nearly in the form of a paraboloid of revolution, *i.e.* the surface formed by a parabola revolving about its axis, has a very strong light in the focus. The rays, therefore, after reflexion go off in a very strong beam of rays very nearly parallel. When this beam has travelled a long distance, its rays, though still much condensed, have spread sufficiently to form a broad beam of strong light: when a ship passes through this beam, it sees the light at the lighthouse, which it would never do, if there was merely the light burning and no mirror behind to reflect its rays.

It should be noticed that, just as light from the focus on reflexion goes off parallel to the axis, so light entering parallel to the axis would, after reflexion, make a beam converging to the focus. So that if we wish to collect a number of rays from the Sun for instance on to one spot, we can do it by placing a parabolic mirror behind the spot, the larger the better, having the spot for focus.

Suppose that we have, as before, a candle in S', and a piece of phosphorus in S; but that the direct heat from S' is kept off by a screen. Place a parabolic mirror $A'B'$ behind S', with its focus in S' and its axis in the line SS'. Place another parabolic mirror AB with its focus in S and its axis in SS'. The rays from S' will come off $A'B'$ and on to AB parallel to the axis: they will therefore converge to S, and will light the phosphorus.

Fig. 15.



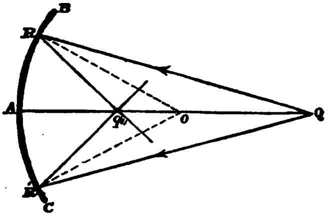
16. A spherical surface is the only one of those which we have named which is easily made, and is constantly used. We will therefore now consider the phenomena connected with reflexion at a spherical surface.

In doing this it will be found better to consider a pencil of rays as a series of conical shells of rays, one within the other, all the vertices coinciding at the luminous point.

Let Q be a luminous point inside the sphere: let O be the centre of the sphere, and let QO produced meet the surface in A. Let the figure be a section made by the plane of the paper, in which both Q and O are supposed to lie.

(i) Consider now a pencil of light of which QOA, which falls perpendicularly on the mirror, is the middle line or axis: then, since QOA is the only ray of the pencil which falls perpendicularly, it is the only ray which will be reflected off perpendicularly,

Fig. 16.



and all the other rays will be reflected symmetrically round it, and it will therefore be the axis of the reflected pencil.

(ii) Of that pencil consider only one of the conical shells of rays, viz. that whose vertical angle is RQR'.

(iii) Of that shell consider only the ray QR.

Join OR: then since BAC is part of a circle RO is the normal at R. Therefore the ray QR is reflected back in direction Rq'', where angle ORq'' = angle ORQ;

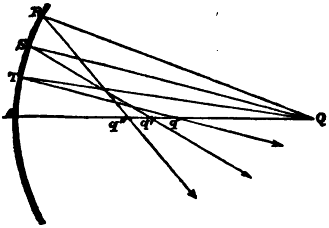
$$\therefore \text{by (Euc. VI. 3)} \frac{QR}{Rq''} = \frac{QO}{Oq''} \text{........(i).}$$

Similarly QR' will be reflected to q'', and so will every ray of the shell whose vertical angle is RQR'. So that the *divergent* conical shell from Q forms, after reflexion, a *convergent* conical shell to q''. The point q'' is called the "*Focus*" of the shell.

Now consider another conical shell whose vertical angle is SQS'. This will in like manner have a focus q' near to q'', and thus each shell will have its own particular focus on the line QA.

17. It can be proved that the nearer to A that the rays from Q fall, *i.e.* the smaller we take the vertical angle of our shell, the more nearly do the foci coincide. That is, qq' is less than $q'q''$; $q'q''$ is less than $q''q'''$, and so on.

Fig. 17.



The point where the reflected rays of the shell with the least imaginable angle cut the axis is called the "*Geometrical Focus*" of the pencil. Close to it then the points $q'q''$ &c. lie thickest, that is, more reflected rays pass close by that point. That is, an eye looking into the mirror would see the strongest light if it happened to be placed so as to catch the rays which before reflexion fall on the mirror very close to the axis. This point is therefore usually taken for the image of Q. If we agree to think only of rays which fall not far away from the axis we can say that, *approximately*, after reflexion they will converge to the point called the "Geometrical Focus," and an eye will therefore see an image of Q at that point. This image then is different from that formed by plane reflexion, in that it is *real*, whereas in plane reflexion it was virtual. But it is not *distinct*, since the rays by which it is seen do not pass accurately through a point, as they appeared to do in plane reflexion.

18. If the student will apply precisely the same reasoning to the case of a convex mirror, he will see that precisely

the same results follow, with this exception—the image will be *virtual* and not real.

The following figures will serve to shew the course of rays to the eye.

Fig. 18.

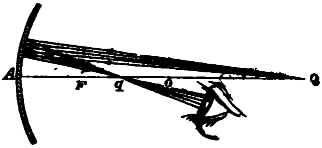
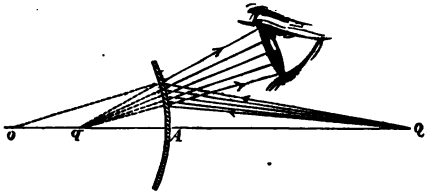


Fig. 19.



19. *To find the position of the "Geometrical Focus" for the concave mirror.*

Since the Geometrical Focus is a point of such importance, it is of great consequence to know exactly where it lies relatively to the object.

In Art. 16 it was proved that $\frac{QR}{Rq} = \frac{QO}{Oq}$, where QR is any ray from Q.

Suppose now that QR is taken close to the axis. Then QR may be regarded as equal to QA; since by bringing the ray closer and closer to the axis, the difference between QA and QR may be made as small as we please. Similarly Rq becomes Aq, where q is *now* the G. F.

Thus

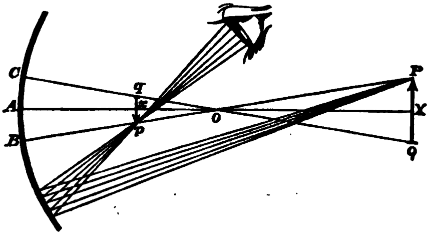
$$\frac{QA}{Aq}=\frac{QO}{Oq} \text{ or } \frac{QA}{Aq}=\frac{QO}{(AO-Aq)}; \quad \therefore \quad \frac{QA}{Aq}=\frac{AQ-AO}{AO-Aq},$$

$$\text{whence } \frac{1}{AQ}+\frac{1}{Aq}=\frac{2}{AO}.$$

Proceeding in a similar way for the convex mirror it would be found that $\frac{1}{AQ}+\frac{1}{Aq}=-\frac{2}{AO}$.

20. Thus finally the image of any point of an object lies on the normal to the mirror from that point, at distance Aq from the point where this normal cuts the mirror, such that $\frac{1}{AQ}+\frac{1}{Aq}=\frac{2}{AO}$.

Fig. 20.



This, as we say, applies for any and every point of the object. Thus an image of any object PXQ will be formed smaller, and inverted.

Thus, in the figure, $\frac{1}{AX}+\frac{1}{Ax}=\frac{2}{AO}$,

$$\frac{1}{BP}+\frac{1}{Bp}=\frac{2}{AO},$$

$$\frac{1}{CQ}+\frac{1}{Cq}=\frac{2}{AO}.$$

Similarly it will be seen that in the convex mirror the image is smaller and erect.

21. The Geometrical Focus of a pencil of rays from an infinite distance—*i.e.* a pencil of parallel rays, is called the "*Principal* Focus" of the mirror.

Now when AQ is infinitely large, $\frac{1}{AQ}$ is indefinitely small and may be left out, and therefore the formula $\frac{1}{AQ}+\frac{1}{Aq}=\frac{2}{AO}$ becomes $\frac{1}{AF}=\frac{2}{AO}$, where F is the principal focus. That is, F is midway between A and O, or the image of a star would be seen there, since a star is a very long way off.

22. There is now one point to notice to which we shall often again refer, regarding the *signs* of lines. We will always make use of the following convention. All lines measured in the direction *opposite* to that of the incident light are to be regarded as positive lines, and *vice versa.* Thus in Fig. 16 AO is positive, and therefore that is taken as the standard case. AQ must, of course, always be positive for divergent pencils. Now consider the case of a *convex* mirror with a divergent pencil. AO is then measured in the same direction as the incident light and is therefore negative. The formula, therefore, which for the concave mirror was $\frac{1}{Aq}+\frac{1}{AQ}=\frac{2}{AO}$ will now be $\frac{1}{Aq}+\frac{1}{AQ}=-\frac{2}{AO}$. (Compare Art. 47.)

This point will present no difficulty to those readers who are familiar with the elements of Analytical Geometry.

Suppose now that we wish to find the position of the *Principal Focus*. This is the Geometrical Focus of a pencil of rays from an infinite distance. Thus if F is the Principal Focus, $\frac{1}{AF}+\frac{1}{AQ}=\frac{2}{AO}$. But $\frac{1}{AQ}=0$, since $AQ=\infty$; $\therefore \frac{1}{AF}=\frac{2}{AO}$; *i.e.* the Principal Focus lies halfway between the mirror and its centre—or the light from a star would make a bright spot there.

23. Thus, finally, we have the following facts connecting Q and q.

(1) Q and q lie on the same normal. For all rays from Q when reflected pass through q, and since the normal ray is reflected along itself, q must lie upon it.

(2) If A be the extremity of the axis, and the pencil be divergent, $\frac{1}{Aq}+\frac{1}{AQ}=\pm\frac{2}{AO}$, according as the mirror is concave or convex.

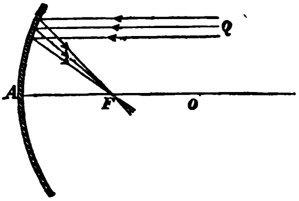
(3) Since $\frac{2}{AO}$ = constant quantity, as AQ increases or decreases, Aq does just the opposite, in concave mirrors; *i.e.* Q and q move in opposite directions. In the convex mirror they move in the same direction.

(4) Since, in the concave mirror, when Q goes farther from A, q comes nearer, and, since, when Q is at its farthest, q comes to F the principal focus, therefore Q and q must lie on the same side of F always. Similar reasoning proves the same fact for the convex mirror.

By a principle which we have used before (Art. 15), if light can travel from any luminous point Q to any other point q, by any number of reflexions, then if a light be placed at q its rays will go to Q by precisely the same path that it used before. This would be the case in fig. 16. Or we can see it at once from the equation $\frac{1}{Aq}+\frac{1}{AQ}=\frac{2}{AO}$, where it makes no difference if we interchange AQ and Aq.

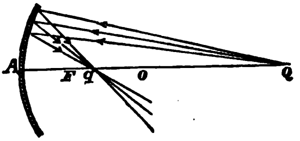
24. We will investigate the motion of Q and q still further.

Fig. 21. (i)



(1) Let F be the principal focus of the concave mirror. Then rays from a point very far from A, that is parallel to the axis, converge very nearly to F (fig. i).

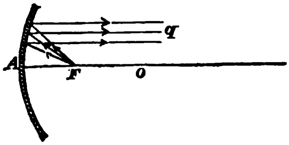
(ii)



(2) Let Q come nearer the mirror from the right. The rays from Q then converge very nearly to some point q to the right of F, since if AQ decrease, Aq must increase (fig. ii).

(3) Let Q come to O the centre. The reflected rays all converge to O. Q and q meet (for if $AQ = AO$ so does Aq).

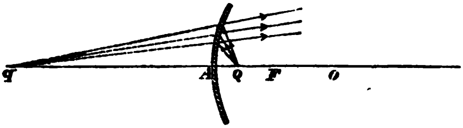
(iii)



(4) As Q goes to the left, Q and q cross, and q goes off to the right, until, when Q has come to F, q has gone to an infinite distance (fig. iii).

(5) As Q still approaches A, q as it were turns the corner at infinity and appears again to the left of A, approaching A (fig. iv).

(iv)



(6) When Q arrives at A, q does so also; for from $\frac{1}{Aq}+\frac{1}{AQ}=\frac{2}{AO}$, if $AQ=0$ so must Aq.

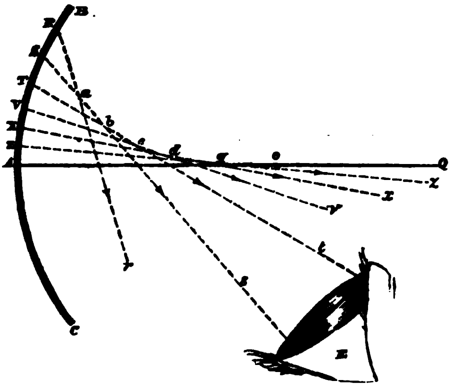
CHAPTER IV.

CAUSTIC CURVES, &c.

25. It cannot be too often recollected that the point *q* is not the point through which *all* the reflected rays pass, for there is no such point. But close to it the rays which fall on the mirror very close indeed to the axis, from *Q*, cut the axis after reflexion. If we take rays from *Q* which fall on the mirror further from the axis, we find that they cut the axis at appreciable distances from *q*; and also that the points of section of the axis become more and more scattered as the rays fall on the mirror further and further from the axis, all being on the side of *q* towards *A*.

Thus, if the dotted lines represent the reflected rays,

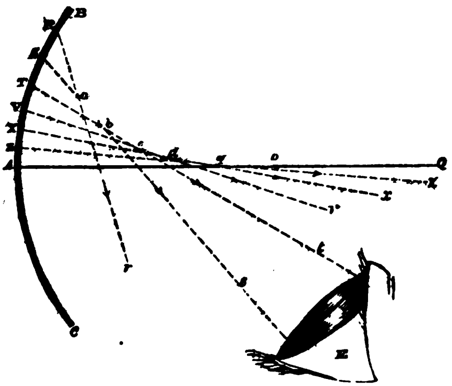
Fig. 22.



the rays before reflexion not being drawn, to save confusion in the figure, fig. 22 will represent the course after reflexion of the rays forming the part of the pencil which strike the mirror above the axis.

The figure, as before, represents a section of the spherical mirror and pencil by the plane of the paper.

Fig. 22.



Let Rr, Ss, Tt... be the reflected rays.

Let Rr, Ss intersect in a,

Ss, Tt .... b,

Tt, Vv .... c, and so on.

Now suppose an eye at E receives the rays Ss, Tt. It receives them as if they both came from b, since they cross one another there. That is, *the eye sees a bright spot there;* brighter than elsewhere along either line, since double light comes from that one spot.

Similarly, if the eye received Rr, Ss, and Tt, it would see two bright spots a and b: for it would receive Rr and Ss

as if from *a*, and *Ss*, *Tt* as if from *b*. And if it received *Rr*, *Ss*, *Tt*, *Vv*, it would see three bright spots *a*, *b*, *c*; and so on.

And if we conceive a great number of rays falling upon and being reflected from the mirror, it is plain that there will lie a great number of spots of light very close together. All these points form a curve, whose cusp or stopping-point is the geometrical focus. Each consecutive pair of rays comes to the eye as if from some one point on this curve; that is, the eye sees points on the curve *abcdq*. Since the pupil of the eye is a very small thing, only a few consecutive rays can enter it, and therefore it can only see a very small piece of the curve at a time.

This curve is called the *caustic* curve.

If we caused *BAC* to revolve about the axis we should obtain a portion of a spherical mirror, but as *BAC* revolved, so would *abcdq*, and therefore, instead of a caustic curve, we should have a caustic *surface*.

26. Figure 23 is intended to represent the caustic curve formed by reflexion from a semicircular mirror of a pencil of rays which fall on it parallel to the axis. The cusp will be at *F*, midway between *O* and *A*; since that is the position of the principal focus. The curve on the paper in the experiment suggested will be brightest at and close to *F*, since the points of intersection are closer there than elsewhere. Or, supposing the rays to come directly to the eye, an eye at *E* for instance will see stronger light than an eye at *E′*, since the rays when they come to *E′* are more spread than those which come to *E*, and therefore fewer of them enter the eye.

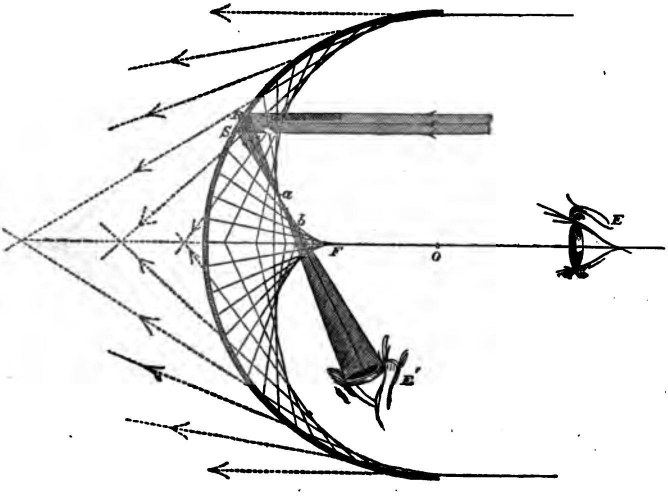
We have hitherto considered the geometrical focus as the image of a luminous point; but that was only when we supposed the eye on or very close to the axis, when the rays passing through the geometrical focus come to the eye. But, clearly, for an eye at E' the image which it sees is not at the geometrical focus but at that point on the caustic curve through which and close to which the rays coming to the eye pass. Thus, if the light of a star was shining on the circular mirror, an eye at E' would see a blurry image of the star at *ab*.

In some cases the caustic curve reduces to only one point: the caustic of a *circle*, the luminous point being in the centre, is the centre. The caustic of an *ellipse*, the luminous point being in one focus, is the other focus; of a *parabola*, for rays coming parallel to the axis from a great distance, it is the focus. The caustic of a plane surface is the point as far behind the surface as the light is in front, on the same normal. In fact it is always a single point when the reflected rays pass or seem to pass *accurately* through it, as is true in all these cases.

27. Since the rays $Rr, Ss, Tt...$ (Art. 25) actually pass through the points $a, b, c...$ the caustic curve exists, although no eye receives the rays. This can be shewn very clearly thus. Bend a piece of watch-spring into an arc of a circle, and fix it edgeways on a sheet of paper. Then place a light in the axis, the least bit above the plane of the paper; or, if the sun is shining, let the rays of the sun fall on it. The reflected rays will fall on the paper and make a brilliant curve, the whole of which is visible to the eye, since it is seen by the light *on the paper*, not by the directly reflected rays. A rough caustic curve may be observed on the bottom

of a tea-cup, formed by the rays which have been reflected from the sides.

Fig. 23.

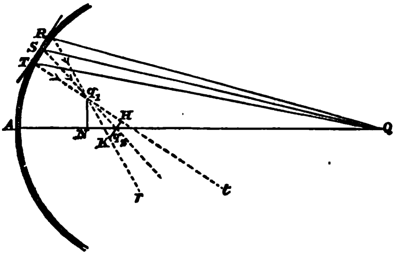


CHAPTER V.

Focal Lines.

28. We have shewn in the last chapter that when a pencil of light is reflected from a spherical surface, the reflected rays of light all touch a certain surface called the caustic surface; or, if we consider only the rays in the plane of the paper, they all touch a curve called the caustic curve, whose cusp is at the geometrical focus of the pencil. For the present we will only consider rays in this plane.

Fig. 24 (i).

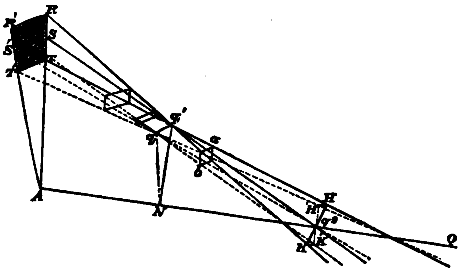


If now we consider only a very few of these rays, all close together, they will, after reflection, touch the caustic curve at points very close together, and if we take them close enough together, we may consider them as passing through only one point on the caustic, but cutting the axis of the direct pencil from Q, of which they form a part, in different points.

Let then QR, QS, QT be rays from Q in the plane of the paper falling on the mirror so close together that after being reflected they may be supposed to go very nearly through one point q_1 on the caustic, and cutting the axis QA of the pencil of which they form a very small part in different points very close together. Let q_2 be the point where the middle ray cuts QA; then all the other rays which fall on the mirror between R and T, pass through the line HK, which we will suppose drawn parallel to the tangent at S.

29. Now suppose the whole figure to rotate through a *very small angle* about the axis AQ, so as to form a solid figure.

Fig. 24 (ii).



Then (i) RT describes a portion of a ring, the shaded part in the figure representing the part on which the rays fall.

(ii) The point q_1 describes a very small arc of a circle, q_1q_1', radius Nq_1, so small that it may be regarded as a very small straight line perpendicular to the plane of the paper; and rays pass all through this small line.

(iii) HK, since it rotates about the point q_2, describes a figure which may be called an elongated figure of eight, so that there are rays passing through all the surface of the two triangles Hq_2H', Kq_2K'. But this figure of eight is so very nearly a straight line, since the angle through which we have turned the figure is so very small, that we may regard it as really such.

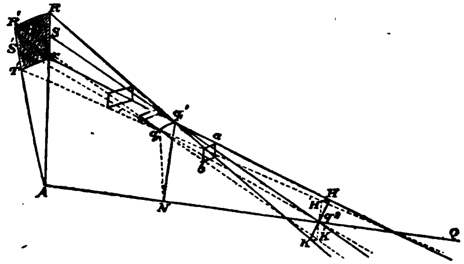
The line q_1q_1' is clearly at right angles to the plane RAQ, since it is formed by the rotation of the whole figure about AQ, which is in that plane.

Thus we may say, approximately, that all our rays of this very small pencil after reflection pass through two very small lines at right angles to one another. The first of these, the one formed by the revolution of q_1, is called the "*Primary Focal Line*," the second is called "*The Secondary Focal Line.*"

(One difficulty that the reader will probably have felt in this subject is the fact that in order that our results may be even approximately true, we are obliged to take only such very small pencils of light. But if he will consider what a very small thing the pupil of his eye is, and what a small pencil of light it can receive, this difficulty will probably vanish.)

30. If a section be taken of the reflected pencil at q_1 by a plane parallel to the tangent plane at S, we have seen that

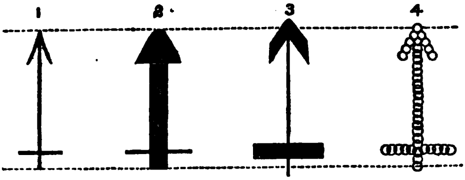
Fig. 24 (ii).



the section is approximately a straight line in that plane *perpendicular to* the plane RAQ. If this plane be supposed to move gradually from q_1 to q_2, remaining parallel to itself, the height of the section increases and the breadth diminishes in the moving plane, until at q_2 the section is a straight line *in* the moving plane, but at right angles to its former direction. Since this change goes on gradually, there must be one position of the plane at which the height and breadth of the sections must be the same, and the section, or the spot of light formed on the plane, is very nearly square. The section of the reflected pencil at this point, *i.e.* ab, may be called the "Section of least confusion" of the pencil, and if the corners be cut off $RTT'R'$, so as to make it oval, this section becomes nearly a circle, which is called the "Circle of least confusion."

Thus it can now be seen that if a screen were placed at q_1 an image of an arrow (fig. 25, 1) placed at Q would be seen thus (fig. 25, 2): while, if placed at the point q_2 the screen would receive an image thus (fig. 25, 3). And, lastly, if placed at the point where the circle of least confusion is formed, it would be seen thus (fig. 25, 4).

Fig. 25.



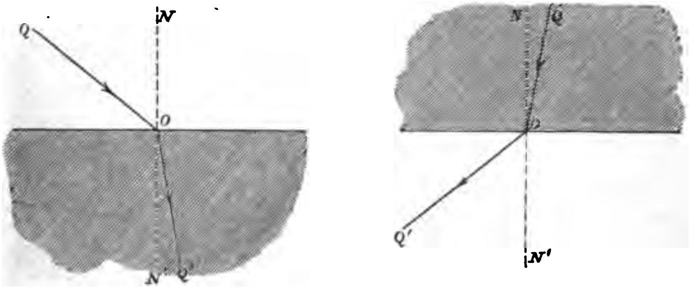
31. Thus, finally, an eye *on the axis* receives a small pencil which may be regarded as diverging approximately from a *single* point called the Geometrical Focus—since for a very small pencil *close to the axis* these two small lines and the circle of least confusion all come together. But supposing the eye not to be in the axis, and therefore not capable of receiving a small pencil which falls close to the axis, we see that we have found two ways of expressing the nature of the image which it sees of the luminous point, viz. either we may regard the pencil which enters the eye as diverging from a point on the caustic surface, or, more accurately still, from two very small lines at right angles to each other. Since the circle of least confusion is the nearest approach to a point that the reflected pencil can reach, we may take *it* as the image of Q, and if Q is of finite size, the image will be as in fig. 25 (4). The size of this circle clearly measures the indistinctness of the image.

CHAPTER VI.

Refraction at a plane surface.

32. (i) When a ray of light has been passing through any one medium of uniform density, and suddenly reaches another medium of different uniform density, it is found that it does not go on in its original line, but is bent aside on entering the new medium. It is also found by experiment, that, if it enters a dense medium from a rare medium, it is turned nearer to the normal, as in (i); if from a dense medium into a rare medium, it is turned further from the normal as in (ii).

Fig. 26.

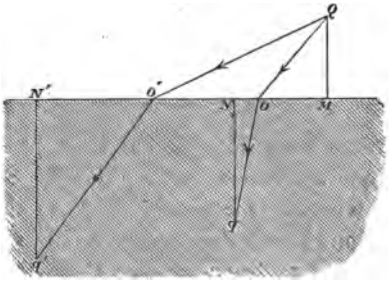


(i) (ii)

(ii) It was also found that the following relation held between the directions of the ray before and after refraction.

Let QOq be a ray in the plane of the paper. Take $Oq = OQ$ and draw QM, qN perpendicular to the surface. Then the relation that holds is this. $OM : ON$ in a constant ratio,

Fig. 27.



whichever ray we take, supposing the media to remain unaltered. Thus, if $QO'q'$ be another ray, then $O'M : O'N'$ in the same ratio as before: if $OM = \frac{3}{2} ON$, then $O'M = \frac{3}{2} O'N'$ and so on.

33. The fact stated in Art. 32 (i) was first distinctly stated by an Arabian mathematician, named Alhazen, who lived about 1100; although the phenomena of refraction had been remarked sooner: Seneca, for instance, remarks that an oar in clear water appears broken, and that apples, seen through a glass are magnified, and Archimedes is said to have published a book "on a Ring seen under water*."

Alhazen also knew that the quantities of refraction differ according to the magnitude of the angle QOM (fig. 27), and moreover that the angles of refraction do not follow the proportion of the angles of incidence, *i.e.* that if, for instance $QOM = 2QO'M$, then qON is not $= 2q'O'N'$ (fig. 27).

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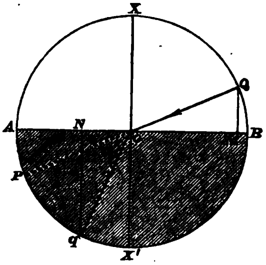
* Whewell's *History of Inductive Sciences*, Vol. II. p. 345.

After reaching this point, there remained nothing to be done with regard to refraction except to go on experimenting and conjecturing till the true law of refraction was discovered, viz. the law in Art. 32 (ii). This was discovered at last by Wildebrord Snell, a Swede, about 1621, but the law was first published by Descartes, who had seen Snell's papers. Snell enunciated the proposition in the following form.

Let Q be the luminous point and QO a ray meeting the surface in O. Join OQ, and describe a circle AQB, centre O, distance OQ. Draw QM perpendicular to the surface,

Fig. 28.



and take $ON : OM$ in the given ratio: draw Nq at right angles to the surface, meeting the circle in q and join Oq, then Oq is the direction of the ray after refraction. The ratio $OM : ON$, *i.e.* the fraction $\frac{OM}{ON}$, is usually denoted by the symbol μ, and is called the *Refractive Index* of the media.

If QO be produced to meet the circle in P, the angle POq, or the difference between the angles QOX, qOX'

which are called the angles of incidence and refraction, is called the *Deviation* of the ray QO.

34. This law is called the *Law of Sines;* and the reason of this is easily seen. For if θ and ϕ be the angles QOX, qOX', *i.e.* the angles of incidence and refraction of the ray QO, then $\frac{OM}{ON} = \frac{\sin\theta}{\sin\phi}$; $\therefore \frac{\sin\theta}{\sin\phi} = \mu$, or $\sin\theta = \mu \sin\phi$.

In precisely the same way, if θ' and ϕ' be the same quantity for another ray, $\sin\theta' = \mu \sin\phi'$.

Thus the laws from which all the phenomena of refraction are deduced may be briefly given:

(i) The incident and refracted rays lie in one and the same plane with the normal to point the surface at the point of incidence, and on opposite sides of it.

(ii) If θ and ϕ have the meanings assigned to them, then $\frac{\sin\theta}{\sin\phi}$ or $\frac{OM}{ON}$ is a constant ratio.

Obs. Since on entering a dense medium from a rare one the ray is bent nearer to the normal; $\therefore \frac{OM}{ON}$ or $\frac{\sin\theta}{\sin\phi}$, (*i. e.* μ) is greater than 1, and *vice versâ.* It should be remarked that when a ray of light strikes any refracting substance such as glass or water, &c. some of the light is always reflected, and the larger the angle of incidence is, the more light is reflected.

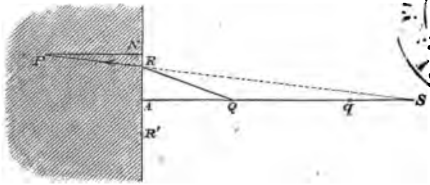
35. Suppose now that a conical shell of rays diverging from Q fall upon the plane refracting surface AB. The rays forming this shell will, by symmetry, all be refracted in just the same manner relatively to the axis QA which is drawn perpendicular to the surface.

Let QR be any one of the rays of our conical shell.

(The figure, as before, represents a section by the plane of the paper.)

QR will be refracted in direction RP, nearer to the normal (by Art. 32). Let RP, produced backwards, meet AQ produced in S.

Fig. 29.



The position assumed by S when QR falls as near as is imaginable to QA is called the geometrical focus of the point Q. Let q be this position.

Take $RP = RQ$ and draw PN perpendicular to the surface.

Then $\dfrac{AR}{RN} = \mu$ (Art. 33); $\therefore AR = \mu . RN$.........(i);

$\therefore \dfrac{AR}{RQ} = \dfrac{\mu . RN}{RQ} = \dfrac{\mu . RN}{RP} = \mu . \dfrac{AR}{RS}$ by similar triangles;

$\therefore RS = \mu . RQ$.

If now we suppose our rays to fall on the surface very close to A, RS becomes $= Aq$,

RQ .... $= AQ$.

$\therefore Aq = \mu . AQ$..........................(ii).

Thus if $\mu = \dfrac{4}{3}$, $AQ = 3$ feet, then $Aq = \dfrac{4}{3} . 3 = 4$ feet.

36. By using trigonometry this investigation may be made much shorter.

For if $RQA = \theta$, $RSA = \phi$, $\dfrac{\sin\theta}{\sin\phi} = \mu$.

But $$\frac{\sin\theta}{\sin\phi}=\frac{RS}{RQ}; \quad \therefore\ RS=\mu RQ,$$

or in the limit $$Aq=\mu AQ.$$

If the refraction is from rare into dense μ is > 1 and q is to right of Q, and *vice versâ.*

37. It is necessary to understand clearly what we have proved in Arts. 35, 36. It is this. Suppose that of all the rays which fall from O upon the surface, we select only a *very small* pencil the axis of which falls at right angles to the surface: then all these rays will after refraction proceed as if they were diverging very nearly indeed from one point q on the axis such that $Aq=\mu\,.\,AQ$. This point then, which is called the geometrical focus, would be, for an eye receiving such rays, the image of the point Q. (Compare Reflexion.)

By the principle that light will always return by the same path as that by which it has gone, we see that a pencil *con*verging to q from the left, would, on coming into the rare medium, be bent so as to converge to Q.

Critical angle.

38. It is found that a ray entering a dense medium from a rare medium is always refracted, however great an angle its direction makes with the normal to the surface, but that there is a direction after passing which a ray entering a medium rarer than its own is not refracted, but is entirely reflected. This we can easily see from fig. 29.

Suppose that there the medium below AB is the denser: *i.e.* $\mu > 1$. Then $OM=\mu\,.\,ON$, *i.e.* OM is always $> ON$. But the greatest value of OM is OB; $\therefore$ ON can never be so large as OB or OA, however near OQ comes to OB. Its greatest value in fact $= \frac{1}{\mu} OB$, corresponding to the ray from Q when Q comes to B.

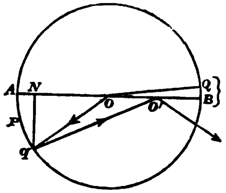
Thus, since light can always retrace its steps, we see that a ray from any point P nearer to A, if $ON = \frac{1}{\mu} OB$, would not be refracted at all, but would be reflected at the surface.

From fig. 30, $qN^2 = Oq^2 - ON^2 = OB^2 - ON^2$.

But $OB = \mu . ON$; $\therefore qN^2 = (\mu^2 - 1)\, ON^2$;

$$\therefore ON = \frac{qN}{\sqrt{\mu^2 - 1}}.$$

Fig. 30.



Therefore, to construct the critical angle for any point q in the dense medium below AB, draw qN perpendicular to the surface, cut off $NO = \frac{qN}{\sqrt{\mu^2 - 1}}$ and join qO; then $NqO =$ critical angle, and Oq is the direction of a ray from Q close to the surface after refraction.

Thus to an eye at q, all objects above the surface would appear to be placed within the cone whose vertical angle $= 2NqO$: while if q be taken as the luminous point all rays such as qO' making a greater incident angle than qO will not get through to the rare medium at all, but will be reflected internally.

Trigonometrical Explanation.

39. If θ and ϕ be the angles of incidence and emergence, $\sin\theta = \mu \sin\phi$, or $\sin\phi = \frac{1}{\mu}\sin\theta$.

Now if the refraction is from *rare to dense* $\frac{1}{\mu} < 1$ and $\therefore \sin\phi$ is < 1 whatever value θ has, *i.e.* ϕ is possible for all values of θ, or a ray can always be refracted into a dense medium.

But if the refraction is *from dense to rare*, $\frac{1}{\mu} > 1$ and $\therefore \sin\phi$ is < 1 only so long as $\sin\theta$ is $< \frac{1}{\mu}$ or $\theta < \sin^{-1}\frac{1}{\mu}$. That is, if the ray falls upon the surface making an angle greater than $\sin^{-1}\frac{1}{\mu}$ it will not be refracted. The ray that makes an angle $= \sin^{-1}\frac{1}{\mu}$ comes out along the surface after refraction.

This angle $\sin^{-1}\frac{1}{\mu}$ corresponds to the angle *NqO* in fig. 30.

Thus, from water into air, no ray will be refracted making an angle greater than $\sin^{-1}\frac{3}{4}$, or $\sin^{-1}\frac{2}{3}$ if from glass into air.

CHAPTER VII.

Miscellaneous.

40. As in reflexion at a plane surface, what was proved true for the single luminous point Q is true of any point of a luminous object. Thus, any point of a luminous object has its Geometrical Focus at a distance from the surface determined by the formula of Arts. 36 and 37.

Let PQX be any luminous object, and BAC the refracting surface denser than that in which is PQX. Then P has its Geometrical Focus at p, where

$$Bp = \mu \,.\, BP.$$

So

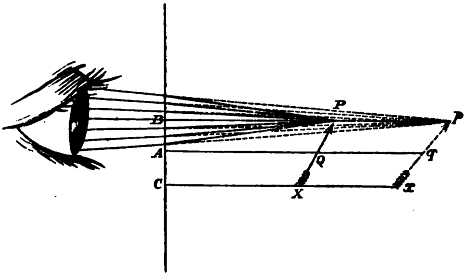
$$Aq = \mu \,.\, AQ,$$

$$Cx = \mu \,.\, CX,$$

PB, QA and XC being supposed normal to the surface.

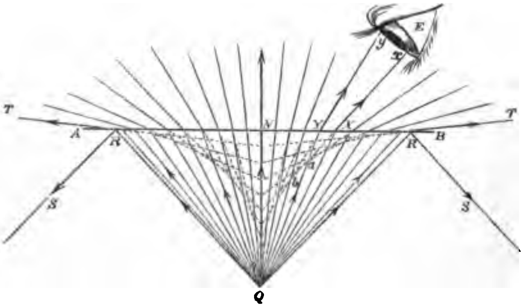
Suppose that PX and px are produced to meet: they will meet in BC produced, since $BP : Bp :: CX : Cx$, and also Bp and Cx are parallel. Now in the corresponding figure for plane reflexion (Art. 9), we were able to take the points p, x, q as the actual images for an eye in *any* position. But here each of these points is to be regarded as the image only when the eye is placed on the axis of that point, so that no eye could possibly see a distinct image pqx of the whole of *PQX*, unless it were at an infinite distance, when it might be regarded as on the axis of all points of the object equally.

Fig. 31.



The image *really* seen of any point *Q* by an eye not on the axis, will be seen in exactly the same way by rays all of which come to the eye as if they touched a caustic curve. Figure 32 represents such a caustic supposing the refraction to take place from dense into rare.

Fig. 32.



The rays *QR* are those which fall upon the surface at the critical angle, and are therefore reflected back again, as

RS, instead of getting through the surface. Thus angle $RQR = 2 \sin^{-1} \frac{1}{\mu}$. A ray falling on the surface the least bit nearer to *N* than *R* will be refracted in direction *RT*, close to the surface. *QN* the normal ray will of course pass through in the same line, and the other rays will all be refracted as in the figure. Now it can be proved, as we said in the corresponding part of reflexion, that if we produce the directions of these rays after refraction backwards, they will cut the axis *QN* in points the consecutive distances of which continually increase.

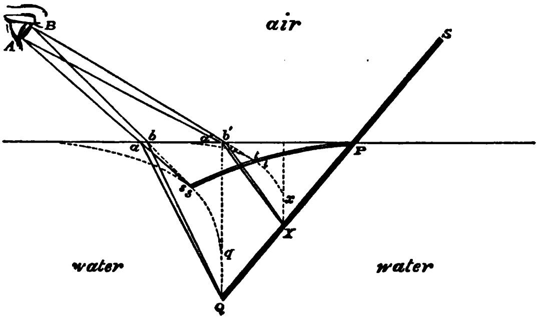
Thus there will be a *virtual* caustic curve formed instead of a *real* one. If *E* then be an eye in any position, and *QXx*, *QYy* be the extreme rays of the very small pencil which come to it, all the rays of this pencil, viz. those falling on the surface between *X* and *Y*, will come to it as if from points on the caustic curve between *a* and *b*, and so a blurry image of *Q* will be seen by the eye *E* at *ab*. This caustic curve, then, has its cusp, as before, in *q* the geometrical focus, and touches the surface at *R* and *R*.

The reader is recommended to draw for himself the caustic formed by refraction from a rare medium into a dense medium.

Figure 33, then, will shew why a stick partly in and partly out of the water appears bent where it enters the water.

Let *Q* and *X* be points of the stick, *q* and *x* their Geometrical foci. Draw the caustic curves of these points *Q* and *X*. Then an eye at *E* will see a blurry image of *Q* at *ss*, and a blurry image of *X* at *tt* where the rays to the eye from *Q* and *X* touch the virtual caustics of those points. The same will be true for every point of the stick. Thus the eye will see the stick as in the figure: the image appearing curved. This effect may be noticed when standing above water of an uniform depth. The bottom appears to slope gradually up to the water as it recedes from the place where we are standing.

Fig. 33.



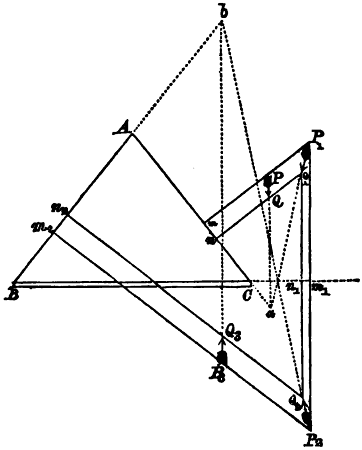
41. We will now give an example which combines the principles of plane reflexion and plane refraction. We shall assume that the image in each refraction is at the Geometrical Focus.

Example. A pencil of light passes through a triangular prism of glass, after having suffered one internal reflexion at one face of the glass. To trace its course from the origin.

Let ABC be a section of the prism made by the plane of the paper. Let BC be silvered, so as to reflect light. Let PQ be the object. Then

(i) By *refraction* at AC, PQ will have an image P_1Q_1 of itself on the normals to AC, such that $mP_1 = \mu . mP$, $nQ_1 = \mu . nQ$. By Art. 44 it will be so inclined as that it meets PQ produced in a on the surface AC.

Fig. 34.



(ii) After entering the glass the light goes as if it came from P_1Q_1 until it falls upon the *reflecting* surface BC, when, by the principles of plane reflexion, it will have its image at P_2Q_2, on the normals to BC, where $m_1P_2 = m_1P$ and $n_1Q_2 = n_1Q_1$.

(iii) After striking BC the light goes as if it came from P_2Q_2 until it emerges at AB, where by refraction it has an image P_3Q_3 such that

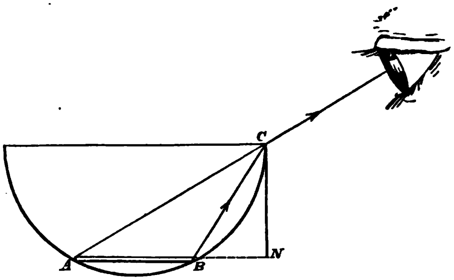
$$M_2P_3 = \frac{1}{\mu} . m_2P_2 \text{ and } n_2Q_3 = \frac{1}{\mu} n_2Q_2.$$

As before, P_3Q_3 and P_2Q_2 would meet in *BA* produced.

It would be a very good exercise for the reader if he would trace the course of a small pencil of rays from *Q* to an eye placed anywhere in the plane of the paper. He must remember that the Geometrical Focus is taken for the image at the refractions, and that at the refractions he must therefore only consider rays falling nearly at right angles.

42. *A coin is placed at the bottom of an empty hemispherical basin of given radius, and is just not visible to an eye looking over the edge. When the basin is filled with water, the whole of the coin is just visible to an eye in the same position. Find the diameter of the coin.*

Fig. 35.



Let *AB* be the diameter of the coin $= 2x$. Since at first it is just not visible to the eye, therefore the eye lies in *AC* produced. Since, when the basin is full it is just visible entirely, therefore the ray from *B* through *C* is just refracted to the eye.

Thus $\theta =$ angle of incidence of $BC = BCN$,

$\phi =$ angle of emergence of $BC = ACN$,

$$\therefore \frac{BN}{BC} = \mu \frac{AN}{AC},$$

where μ = index of refraction from water into air;

$$\text{i.e.} \frac{a-x}{\sqrt{(a-x)^2 + a^2 - x^2}} = \mu \cdot \frac{a+x}{\sqrt{(a+x)^2 + a^2 - x^2}},$$

or
$$\frac{a-x}{\sqrt{2a(a-x)}} = \mu \cdot \frac{a+x}{\sqrt{2a(a+x)}};$$

whence
$$x = a \cdot \frac{1-\mu^2}{1+\mu^2}.$$

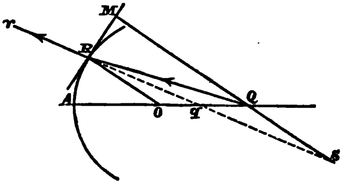
This problem was explained as early as A.D. 140 by Ptolemy, the Alexandrian astronomer.

CHAPTER VIII.

Refraction at a spherical surface.

43. To find the Geometrical Focus of a pencil of rays after direct refraction at a concave spherical surface.

Fig. 36.



Let Q be the luminous point, and let O be the centre. Then QOA is the axis of the pencil from Q which falls directly on the surface. Of the pencil consider any ray QR which falls on the surface at R; let Rr be its direction after refraction, nearer to the normal OR than before. Let RM be the plane touching the spherical surface at R, (the whole figure being, as usual, a section by the plane of the paper): let QM be a perpendicular to this plane. Let MQ and rR produced backwards meet in S.

Since RM is a tangent plane at R, the refraction which takes place there is the same as would take place at the plane refracting surface RM.

$$\therefore\ MS = \mu \,.\, MQ \text{ by (Art. 39. i.)}$$

But, by similar triangles, ROq, SQq,

$$\frac{QS}{Qq} = \frac{OR}{Oq},$$

or
$$\frac{MS - MQ}{Qq} = \frac{OR}{AQ - AO}.$$

Substituting for MS

$$\frac{MQ\,(\mu - 1)}{AQ - Aq} = \frac{AO}{Aq - AO}.$$

In the limit, when R moves up to A, MQ becomes AQ,

$$\therefore\ \frac{(\mu - 1)\,.\,AQ}{AQ - Aq} = \frac{AO}{Aq - AO},$$

whence
$$\frac{\mu}{Aq} - \frac{1}{AQ} = \frac{\mu - 1}{AO} \text{..............(1).}$$

Thus, if a number of rays from Q fall *on the surface very nearly* at A, they will, after refraction, proceed as if they came almost exactly from q, where the position of q is found from the formula (i).

Example. $\mu = \frac{3}{2}$, $AQ = 6$ feet, $AO = 4$ feet,

$$\frac{3}{2}\,\frac{1}{Aq} = \frac{1}{8} + \frac{1}{6} = \frac{7}{24},$$

$$\therefore\ AQ = \frac{36}{7} = 5\tfrac{1}{7} \text{ feet.}$$

If the surface were convex the formula would be found to be

$$\frac{\mu}{Aq} - \frac{1}{AQ} = -\frac{\mu - 1}{AO}.$$

44. If now we suppose Q to be very distant, a star, for instance, i.e. if we suppose a pencil of *parallel* rays to fall on the surface close to A and parallel to the diameter through A, we may in our formula neglect $\frac{1}{AQ}$, and we have then

$$\frac{\mu}{AF}=\frac{\mu-1}{AO} \text{ or } AF=\frac{\mu}{\mu-1}AO,$$

where F is the position which the point q assumes in such a case. As in reflexion, F is called the "*Principal Focus.*" Thus for parallel rays from air into water,

$$AF=\frac{\frac{4}{3}}{\frac{4}{3}-1}AO=4.AO.$$

On the whole then we shall have, as in the corresponding case of reflexion, the following rules for determining the position of our Geometrical Focus.

(i) Q and q must lie on the same normal, since all the rays from Q are supposed to proceed as if from q, after refraction, and since the direction of the normal ray is not altered at all.

i.e. The Geometrical Focus of any point must lie on the normal to the surface from that point.

(ii)
$$\frac{\mu}{Aq}-\frac{1}{AQ}=\pm\frac{\mu-1}{AO}$$

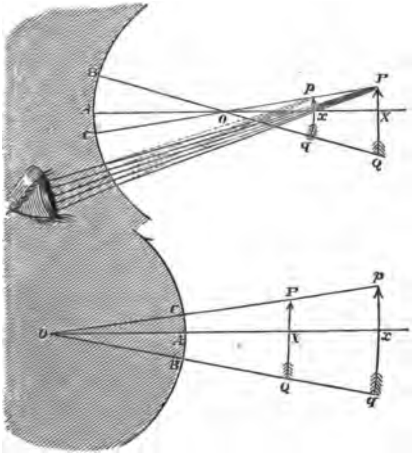
according as the surface is concave or convex.

(iii) Q and q move in the same direction, since as AQ increases, so must Aq from the formula.

(iv) Hence Q and q always lie on opposite sides of the principal focus.

45. Let PQX be an object in front of the spherical refracting surface BAC.

Fig. 37.



By the last article the Geometrical Focus of P is at p, fig. 38 (i), where

$$\frac{\mu}{Cp} - \frac{1}{CP} = \frac{\mu - 1}{CO},$$

that of Q is at q where

$$\frac{\mu}{Bq} - \frac{1}{BQ} = \frac{\mu - 1}{BO},$$

that of X is at x where

$$\frac{\mu}{Ax} - \frac{1}{AX} = \frac{\mu - 1}{AO}.$$

In fig. (ii).

$$\frac{\mu}{Cp} - \frac{1}{CP} = -\frac{\mu - 1}{CO},$$

$$\frac{\mu}{Bq} - \frac{1}{BQ} = -\frac{\mu - 1}{BO},$$

$$\frac{\mu}{Ax} - \frac{1}{AX} = -\frac{\mu - 1}{AO}.$$

46. There is one more point to notice. In the formulæ for concave and convex refracting substances, we observe that the only difference is in the sign of the right-hand side of the equation. The two formulæ will coincide, if we make this assumption about the *signs* of our lines. All lines measured from the origin A *opposite* to the direction in which the light is travelling are to be regarded as *positive*, and *vice versâ*.

Applying this to figure 36, we see that all the lines there drawn from A are opposite to the direction of the incident light, and are all therefore positive. This case then may be regarded as the standard case.

But if the reader will draw the corresponding figure for a convex substance he will see, as in fig. 37 (ii), that AO is drawn from A in the negative direction. This accounts for the difference in sign in the two formulæ.

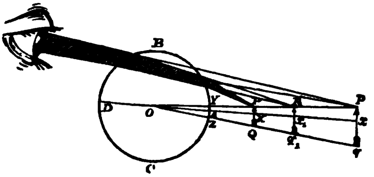
47. It is sometimes more convenient to have a formula referred to the centre as origin, instead of one referred to A.

From figure 36, $Aq = AO + Oq$, $AQ = AO + OQ$.

Substitute in the formula $\frac{\mu}{Aq} - \frac{1}{AQ} = \frac{\mu - 1}{AO}$, and from it we obtain this $\frac{1}{Oq} - \frac{\mu}{OQ} = \pm\frac{\mu - 1}{AO}$, for concave or convex surface. Fig. 36 is clearly the standard case for the formula origin O, since then all the lines OA, OQ, Oq are positive.

48. We will, as a good example, trace the effects of refraction through a sphere of some substance denser than the surrounding medium.

Fig. 38.



We will suppose that the rays after each refraction come as if accurately from the Geometrical Focus.

(i) Let a small pencil of rays from the point P of the object PQ fall upon the surface BAC and be refracted into the sphere. After entering the sphere these rays will proceed as if they came from p_1 on the normal Op_1 where

$$\frac{1}{Op_1} - \frac{\mu}{OP} = -\frac{\mu - 1}{AO} \quad \ldots\ldots\ldots (A).$$

(ii) These rays, which come as if from p_1, will, on being refracted out into the original medium, proceed as if they came from p, where, since the refraction is from dense to rare,

$$\frac{1}{Op} - \frac{\frac{1}{\mu}}{Op_1} = \frac{\frac{1}{\mu} - 1}{AO},$$

$$\text{or } \frac{1}{Op_1} - \frac{\mu}{Op} = \frac{\mu - 1}{AO} \quad \ldots\ldots\ldots (B).$$

To trace the course of the rays through the sphere to the eye, we do just as in reflexion,

(i) Join the last image to the eye by straight lines and shade from the eye to the last surface.

(ii) Join the points where these lines cut the last surface, to the last image but one, and shade to the last surface but one.

(iii) Join these last points to the object and shade.

The shaded part represents the rays from the point P to the eye.

Subtracting (B) from (A)

$$\frac{\mu}{Op} - \frac{\mu}{OP} = -\frac{2(\mu - 1)}{AO},$$

or

$$\frac{1}{Op} = \frac{1}{OP} - \frac{2(\mu - 1)}{\mu . AO}.$$

49. The three following examples are recommended to the reader's attention.

(i) The last example, supposing that instead of a sphere, the refracting solid is a hemisphere, and that the light falls first upon the convex surface.

(ii) A glass sphere is divided into two hemispheres, the index of refraction in one being different from that in the other. Find the positions of the images as in Art. 48.

(iii) An eye is placed close to the surface of a sphere of glass ($\mu = \frac{3}{2}$) which is silvered at the back. Shew that the image which the eye sees of itself is $\frac{3}{5}$ natural size.

CHAPTER IX.

Refracting Plates and Refracting Prisms.

50. *A Refracting Plate* is a portion of a refracting medium bounded by two parallel plane surfaces, as a pane of glass.

Suppose a ray of light enters and emerges from a refracting plate. Then if we recollect the principle that light would exactly retrace its steps and that the sides of the plate are parallel, we see that its direction on emergence must be exactly parallel to its direction before entering the plate. If a ray passing through any number of plates placed in contact come to a plate composed of its original medium, its course in this plate will be parallel to its original course.

By means of this experimental fact and the principle quoted it can be proved that if

μ_1 = index of refraction from medium *A* into medium *B*,

μ_2 = ...*A*................*C*,

then $\frac{\mu_2}{\mu_1}$ = index from *B* into *C*.

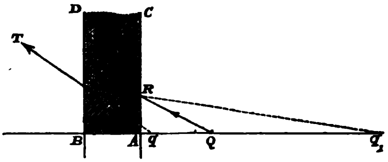
Recollecting that $\mu = \frac{\sin\theta}{\sin\phi}$, the reader can easily prove for himself that if $\mu_1\ \mu_2\ \mu_3 \ldots\ldots \mu_n$ be the indices of refraction from A into B, B into C, C into D...... and if $\mu =$ index from A into the last, then

$$\mu = \mu_1 \, \mu_2 \, \mu_3 \ldots\ldots \mu_n.$$

51. To find the geometrical focus of a pencil of rays refracted through a plate (index $= \mu$).

Let AC and BD be the sides of the plate: let $QRST$ be the course of a ray. After refraction *into the glass* at the first surface at R we have the rays diverging from a focus q_1,

Fig. 39.



where

$$Aq_1 = \mu \, . \, AQ \ldots\ldots \text{(i)}$$

Art. 36 (ii). The rays now form a diverging pencil in the glass, from q_1: therefore after refraction *out of the glass* at S into the air, they diverge as if from a focus q, where

$$Bq = \frac{1}{\mu} Bq_1 \, ;$$

$$\therefore Bq_1 = \mu \, . \, Bq \ldots\ldots\ldots\ldots \text{(ii)}.$$

Subtracting (i) from (ii)

$$AB = \mu \, . \, AB + \mu \, . \, Aq - \mu \, . \, AQ,$$

$$\therefore Aq = AQ - \frac{\mu - 1}{\mu} AB.$$

Here we have assumed as usual, what is only approximately true, that the rays come accurately from the Geometrical Focus.

52. It can be shewn that when a ray is refracted out of one medium into another, as the angle of incidence increases, so also does the *Deviation*. Art. 34. This principle, which admits of an easy Geometrical proof, will be of great use in our investigations into the properties of prisms.

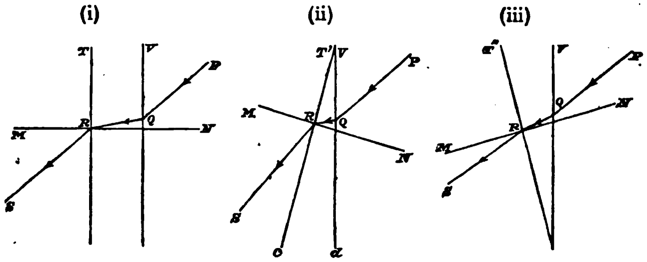
A *prism* is a portion of any refracting medium, bounded by two plane surfaces inclined at any angle to one another.

The inclination of these faces to one another is called the "*Refractive Angle.*"

The line in which these faces meet is called the "*edge*" of the prism.

We will now prove that "*A ray passing through a prism of a medium denser than the surrounding medium is always turned from the edge of the prism.*"

Fig. 40.


(i) (ii) (iii)

(i) Let *PQRS* (fig. 40, (i)), be a ray passing through a plate bounded by parallel sides *TR*, *VQ*, and let *RS* be its direction after emergence.

(ii) By turning *RT* round into the position *RT′*, we make our plate into a prism whose edge is turned towards *T′V*. The angle which *QR* makes with the normal has now increased, therefore the deviation has increased, *i.e.* the angle which RS makes with the normal has increased still more, and the ray therefore emerges as RS' nearer to the thick end of the prism than PQ. If the deviation did not increase it would emerge just parallel to PQ, *i.e.* it would not be bent at all.

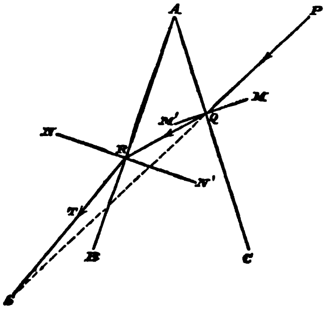
(iii) Next we can turn our plate into a prism with the thick end towards T and N. The angle which QR makes with the normal has now decreased, therefore the angle which the emergent ray makes with the normal has now decreased still more, therefore the ray comes out as RS'' nearer again to the thick end.

Thus in every case the emergent ray is turned from the edge.

53. The result of the last Article enables us to prove that "A ray passing through a prism is always turned to the edge of the prism, when the medium of which the prism is composed is denser than the surrounding medium."

54. We will now find the value of the deviation of a ray after refraction through a prism.

Fig. 41 (i).



(i) Let $PQRS$ be the course of a ray: let the original and new directions of the ray meet in S.

Then $\textit{Deviation} = R\hat{S}Q = R\hat{Q}P - SRQ = RQA + 90^\circ$ $- PQM - (90 - \hat{S}RN + A + \hat{R}QA)$

$$= T\hat{R}N - P\hat{Q}M - A$$

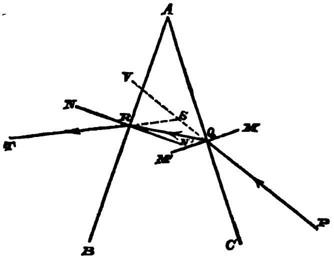
= angle of emergence − angle of incidence − refractive angle of prism for rays above the normal.

(ii) $\textit{Deviation} = TSV = 180 - RSQ = 180 - T\hat{R}Q + S\hat{Q}R$

$$= 180 - 90 + TRN - A - RQA + (RQA - 90 + PQM)$$

$$= TRN + PQM - A$$

Fig. 41 (ii).



= angle of incidence + angle of emergence − angle of prism for rays below the normal.

55. Now it can be proved that when a ray makes the angle of emergence equal to the angle of incidence, the deviation is less than for any other ray.

If then D = deviation, ϕ and ψ be the angles of incidence and emergence, and A the refractive angle of the prism,

$D = \phi + \psi - A$. When therefore the ray has minimum deviation, $D = 2\phi - A$.

And if ϕ' and ψ' be the angles of refraction at the first, and incidence on the second surface, $\phi' + \psi' = A$, or here $2\phi' = A$;

$$\therefore \mu = \frac{\sin\phi}{\sin\phi'} = \frac{\sin\dfrac{D+A}{2}}{\sin\dfrac{A}{2}} \quad\ldots\ldots\ldots\ldots\ldots(1).$$

From this we can determine the refractive index of any substance which can be made into a prism. Measure the minimum deviation D, and the refractive angle A and substitute in (1).

The fact stated at the beginning of this Article can be proved either by the aid of Trigonometry (Parkinson's *Optics*, Art. 94), or Geometrically (*Cambridge Messenger of Mathematics*, Nov. 1869).

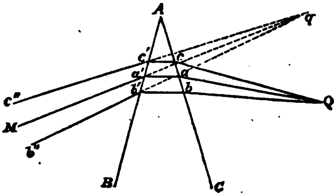
56. The rays close to the ray of minimum deviation on either side have smaller deviation than those more remote. That is, on emerging at the other side of the prism, such rays would form a more compact beam than those elsewhere, that is, an eye placed so as to catch such beams would see stronger light than an eye elsewhere.

This is of the very greatest importance in the discussion for instance on the causes of the Rainbow.

We can deduce a convenient method of determining the position of minimum deviation, i.e. the position in which the prism must be placed so that our eye may receive the rays close to that of minimum deviation, practically. Prick a small hole in a sheet of paper, and observe the spot of light through the prism. Then turn the prism about in the plane perpendicular to its edge: as you turn it one way you will see the image move upwards; as you turn it the other, it will move down. But in one certain position it will move down whichever way the prism is turned. This position then is the position of minimum deviation: since the meaning of the experiment is, that on passing this position either way the deviation increases.

57. We have said that the rays which form the very small beam close to the ray which has minimum deviation emerge so that their deviations have been very nearly the same. Since we are considering only a very small beam we may suppose that $b'c' = bc$. If then the deviations of the three rays in the figure were exactly the same, the emergent rays would be just as much inclined to one another as the incident rays were, and therefore, since $b'c' = bc$ practically, they would meet at q where $a'q = aQ$.

Fig. 42.



Now the deviations are so very nearly the same, and $b'c'$ is so very nearly $= bc$, if we let the rays fall close to the edge of the prism, that the emergent rays of this small beam do very nearly indeed emerge as if from a single point.

If we take a prism with a very small angle A, this will, of course, be still more nearly true.

58. We know that deviation $= \phi + \psi - A$

where
$$\sin\phi = \mu \sin\phi',$$
$$\sin\psi = \mu \sin\psi'.$$

Suppose the rays incident on a prism with a small refractive angle very nearly perpendicular to the first surface.

Then
$$\left.\begin{array}{l}\phi = \mu\phi \\ \psi = \mu\psi\end{array}\right\} \text{very nearly;}$$

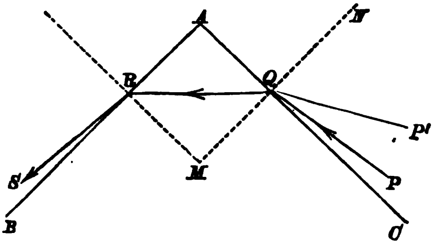
since ϕ' and ψ' are so very small, and therefore
$$\left.\begin{array}{l}\sin\phi' = \phi' \\ \sin\psi' = \psi'\end{array}\right\} \text{very nearly;}$$
$$\therefore \phi + \psi = \mu(\phi' + \psi') = \mu A;$$

$\therefore$ deviation $= (\mu - 1) A$ very nearly $=$ constant quantity, i. e. the deviations for the rays making the small beam are the same, and therefore the same result follows as in last Article.

59. Suppose that we wish to construct a prism such that no ray shall be able to get through it.

Let ABC be a prism such that a ray from P, which enters the prism as nearly as possible parallel to AC, comes out as RS, as nearly as possible parallel to AB.

Fig. 43.



Then, clearly, each of the angles *MQR*, *MRQ* is the critical angle of the medium.

But $M\hat{Q}R + M\hat{R}Q = A$, i.e. 2 . critical angle $= A$.

Now suppose a ray came from P', so that angle $P'QN$ is $< PQN$. Then angle *MQR* is decreased, and therefore *QRM* is increased, i.e. the ray *QR* makes an angle with the normal *AB* greater than the critical angle, and will therefore not be refracted out at *R*, but will be internally reflected.

Therefore the prism through which no ray can pass is that where the refractive angle of the prism = twice the critical angle of the medium.

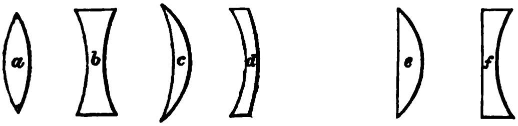
CHAPTER X.

Lenses.

60. A lens is a portion of a refracting medium bounded by two spherical surfaces: the straight line joining their centres being called the *axis* of the lens. This definition includes those lenses in which one surface is plane, since a plane may be regarded as a sphere of an infinite radius.

Lenses are distinguished by different names, according to the nature of their surfaces. In the following figures the light is supposed to pass from right to left. Thus there are six kinds of lenses; or, including a prism and a plate under the definition, there are eight.

Fig. 44.



Thus (*a*) is called a *double convex* lens.
(*b*) is called a *double concave* lens,
(*c*) and (*d*) are called *convexo-concave* lens,
(*e*) is called a *convexo-plane* lens,
(*f*) is called a *concavo-plane* lens.

61. One of the earliest notices of a lens is in the writings of Seneca, A.D. 64 (*Nat. Quest.* lib. 1. ch. 7), in which he states the fact that objects seen through a ball of glass full of water appear larger. This has already been explained (Art. 49). Pliny in A.D. 79, speaking of the manufacture of glass at Sidon, says, "siquidem etiam *specula* excogitaverat." This word "*specula*" probably refers to lenses, as Pliny states that the art of grinding glass was then known.

After an interval of some centuries we arrive at the time of Alhazen, the Arabian mathematician, A.D. 1100. He has attempted an anatomical description of the eye, and has assigned the respective functions of the several parts of vision. To do this as he has he must, as will be seen, have had a good general idea of the effect of lenses.

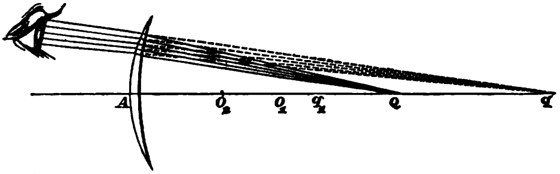
Jean Baptiste Porta of Naples, A.D. 1445, states that if a small orifice be made in the shutter of a room from which all other light is excluded, and if a small convex lens be placed at the aperture, external objects will appear on a sheet of paper, placed at the proper distance, so distinct as to be instantaneously recognized. We will now explain this latter fact. It must be noticed that although the facts were noticed, and could in a rough way be explained, yet until the discovery of the law of refraction by Snell in 1621, a real explanation was wanting.

62. A pencil is said to be *directly* refracted through a lens when the axis of the pencil is coincident with the axis of the lens.

The process of finding the geometrical focus of a pencil after direct refraction through a lens is precisely similar to that used for a plate (Art. 52).

Let Q be the point of light from which a pencil falls directly on the lens. The axis of the pencil is therefore QAB, a common diameter of the spherical surfaces.

Fig. 45.



Let (r) and (s) be the radii of the spherical surfaces (A) and (B). Let q_1 and q be the geometrical foci of the pencil after first and second refractions, the first being *into* glass, the second *out* of it.

Let O_1 and O_2 be the centres of the surfaces (A) and (B). Then q_1 and q will have positions as in the figure.

From the *first* refraction we obtain

$$\frac{\mu}{Aq_1} - \frac{1}{AQ} = \frac{\mu - 1}{r},$$

from the *second*,

$$\frac{\frac{1}{\mu}}{Bq} - \frac{1}{Bq_1} = \frac{\frac{1}{\mu} - 1}{s} \text{ or } \frac{1}{Bq} - \frac{\mu}{Bq_1} = -\frac{\mu - 1}{s}.$$

If the thickness of the lens is so small that Aq_1 may be regarded as equal Bq_1 and $Aq = Bq$, then by adding these equations

$$\frac{1}{Aq} - \frac{1}{AQ} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right),$$

which determines the position of q. As before, Q and q are called "*conjugate foci*" with respect to the lens.

Suppose AQ to be very large, i.e. suppose the rays to fall parallel, and let q then come to F. Then

$$\frac{1}{AF} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right).$$

The point F is *then* called the "*principal focus*" of the lens and AE its "*focal length*." This is usually written f. Thus

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{s}\right); \therefore \frac{1}{Aq} - \frac{1}{AQ} = \frac{1}{f},$$

where q is the geometrical focus for any point Q.

If r and s are equal

$$\frac{1}{f} = \pm \frac{2(\mu - 1)}{r}$$

for *double concave or convex.*

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63. It should be noticed that we have found this equation from the case of a *concavo-convex* lens, that being taken as the standard case because the quantities r and s there are both positive. If we took the case of a *double convex* lens the formula would be

$$\frac{1}{Aq} - \frac{1}{AQ} = (\mu - 1)\left(-\frac{1}{r} - \frac{1}{s}\right),$$

found from the first by putting $-r$ for $+r$ by the principles in Art. 22.

For a concavo-plane lens since s is then $= \infty$

$$\frac{1}{Aq} - \frac{1}{AQ} = (\mu - 1)\frac{1}{r}.$$

For a convexo-plane lens since s is then $= \infty$

$$\frac{1}{Aq} - \frac{1}{AQ} = -(\mu - 1)\frac{1}{r}.$$

For a double concave lens

$$\frac{1}{Aq}-\frac{1}{AQ}=(\mu-1)\left(\frac{1}{r}+\frac{1}{s}\right).$$

Obs. In any given lens the position of the focus is the same whichever face be turned towards the incident light. For, neglecting the thickness of the lens, our formula is

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right).$$

Now if we turn our lens round, the formula must be written

$$\frac{1}{f'}=(\mu-1)\left(\frac{1}{(-s)}-\frac{1}{(-r)}\right)=\frac{1}{f}.$$

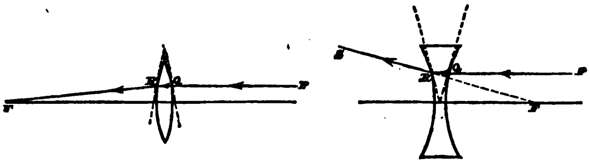
64. If we were told the nature of our lens, i.e. the radii of the surfaces which formed it, we could from

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{r}-\frac{1}{s}\right)$$

determine on which side of the lens the principal focus lay, by noticing the *sign* of the right-hand quantity. But a better rule can be given thus. If the lens be thickest in the middle, the principal focus is on the side away from the incident light, i.e. focal length is negative. If the lens be thinnest in the middle, the reverse is the case. It can be proved thus.

Let tangent planes be drawn at Q and R: then the course of the ray is the same as it would be through the prism formed by these planes: and therefore, since the thicker part of the lens is in the same direction as the thicker part of the prism, and since a ray is bent towards the thicker part of the prism (Art. 53), therefore the ray is bent towards the thicker part of the lens. Therefore the position of the principal focus must be as given in the rule.

Fig. 46 (i). (ii)



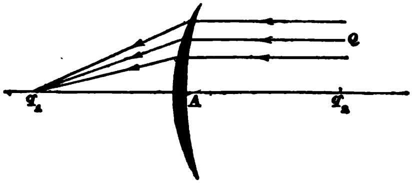
The reader may feel some difficulty in reconciling this with fig. 45. The next Article will, however, explain it.

65. We will investigate this equation $\frac{1}{Aq} - \frac{1}{AQ} = \frac{1}{f}$ in just the same manner as we did in the case of spherical reflexion.

(1) Since f= constant quantity, as AQ increases or decreases, so must Aq, i.e. Q and q always move in the same direction.

Let q_1 be the principal focus of the lens for rays coming from the right. Take $Aq_2 = Aq_1$; then q_2 is the principal focus for rays from the left (*Obs.* Art. 63).

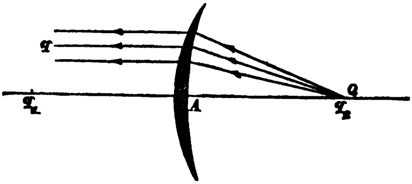
Fig. 47 (i).



As Q moves, from an infinite distance to the right, towards A, q_1, which was at q, moves off to the left (Art. 64).

(2) When Q comes to q_2, the image q has moved off to the left to an infinite distance, since q_2 is the principal focus for rays from the left. (ii)

(ii)

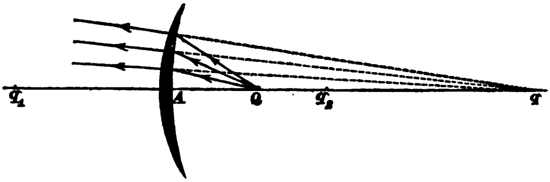


(3) As Q still approaches (A) the image q as it were turns the corner at infinity, and appears again to the right of q_2. (iii)

(q is now only a *virtual* image.)

This is the case taken for fig. 45.

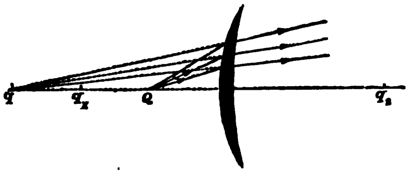
(iii)



(4) Q and q approach A together, and arrive there simultaneously: for, when $AQ = 0$, so does Aq.

(5) As Q goes off to the left, so does q. (iv)

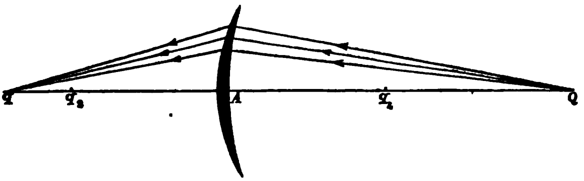
(iv)



(6) And, lastly, when Q arrives at q_1, q has again gone away to an infinite distance.

Figure (v) shews the usual case where Q is a considerable distance from A; intermediate between figs. (i) and (ii).

(v)



This then is the explanation which we promised in Art. 61 of Porta's experiment. The reader is strongly advised to study this Article thoroughly, and to work out the same investigation for a double concave lens.

66. If the focal length of a lens is small, the parallel rays from the object are clearly more bent by the lens than when the focal length is large. That is, since f is the focal length, $\frac{1}{f}$ is a measure of the amount that the rays are bent by the lens. It is therefore called the "*Power*" of a lens.

Suppose now that we had three lenses C_1, C_2, C_3 with a common axis. Let Q be a luminous point on the axis, and let q_1, q_2 and q be the geometrical foci of the refracted pencil at these lenses respectively.

Then from the *first* refraction $\frac{1}{C_1q_1} - \frac{1}{C_1Q} = \frac{1}{f_1}$.

Then from the *second* refraction $\frac{1}{C_2q_2} - \frac{1}{C_1q_1 + a_1} = \frac{1}{f_2}$.

Then from the *third* refraction $\frac{1}{C_3q} - \frac{1}{C_2q_2 + a_2} = \frac{1}{f_3}$,

where a_1 a_2 are the distances between C_1 and C_2, and between C_2 and C_3, and f_1, f_2 and f_3 are the focal lengths of the lenses.

If we eliminated C_1q_1 and C_2q_2 between these equations we could find C_3q. For instance, if $a_1 = a_2 = 0$, i.e. if the lenses are in contact and very thin, we have by addition

$$\frac{1}{C_3q} - \frac{1}{C_1Q} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}.$$

Thus the power of the three lenses = sum of the powers of the three lenses separately. Or, instead of the three lenses we might have one lens such that if F were its focal length, then $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$, and the effect would be the same. Similarly for any number of lenses, except that then, even for rough calculations, we could not neglect the thicknesses.

67. The focal length of a lens can be found experimentally thus.

The rays from the sun, if allowed to fall directly on a lens, would, since they may be called parallel, form after refraction a convergent conical beam to the principal focus. They do not all pass accurately through one point, but if the beam be intercepted by a screen parallel to the plane of the lens, the little circle of light formed on the screen will be smallest at some one position of the screen. This little circle is to be taken as the principal focus. Measure the distance from the lens to the screen, and the focal length is then found.

There is one other remark we wish to make on the appearance of an object as seen through a double concave lens. If, after reading the reasoning and looking at the figures which shew that an image is formed nearer to the eye than an object really is, the student takes a double concave lens in his hand, he will be surprised to see that the image is apparently a *very long way off.* The following explanation may perhaps suffice.

(1) He knows the real size of the object before looking at it through the lens;

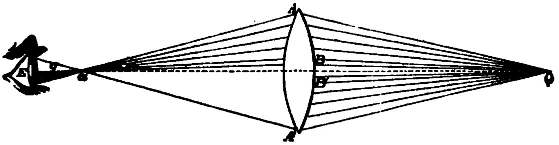
(2) He knows that if an object of a given size is far off it looks smaller than when near;

(3) He sees that the image is small compared to the known size of the object;

therefore his mind at once says that the image is formed a long way off. On looking through the lens for some time, however, this delusion entirely disappears.

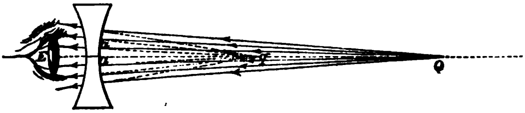
On looking at any object through a double concave and a double convex lens successively, one will notice that the image is far more distinct in the double concave lens than in the double convex: and the reason of this is easily seen if we recollect what has been our fundamental supposition in every one of our investigations; viz. that in asserting that an image is formed at a point which we call the "Geometrical focus," we consider those rays, and those rays *only*, which fall on the surface or lens close to the axis. Rays that come to the eye from the outer parts of the lens tend to make the image indistinct.

Fig. 48.



Thus, in the double convex lens, an eye E would, since it catches all the rays that come from the lens, see an image not quite distinct: for the rays from A and A' cut the axis at a, whereas from B and B' they cut in q. It would see the best image at q, since, as we have said when treating on caustics, the rays are thickest about there.

Fig. 49.

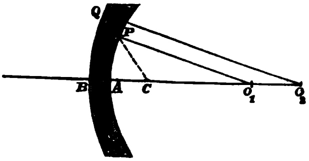


But in the double concave lens, since the rays *spread* after refraction, an eye of the same size as before, even if put close to the lens, will catch only those rays which fall very close to the axis, between a and a', and will therefore see a very distinct image at q.

68. Centre of a lens.

If we join the points of two circles where the radii of the circles are parallel, the joining line when produced will always pass through a fixed point on the line joining their centres.

Fig. 50.



Thus if P and Q be *any* two points on the spherical surfaces such that the radii O_1P, O_2Q are parallel, QP will pass through some one point C on the axis. This point C is called the "*centre*" of the lens.

Now

$$\frac{CO_1}{CO_2} = \frac{O_1P}{O_2Q} \text{ i.e. } \frac{AO_1 - AC}{BO_2 - BA - AC} = \frac{AO_1}{BO_2},$$

whence

$$AC = \frac{AO_1 \,.\, BA}{BO_2 - AO_1}.$$

We see from this value of AC that the position of C is always the same for the same lens, but depends upon the shape of the lens.

(1) If the curvatures are in the same direction, i.e. if the centres are both to the right or both to the left, AC is positive or negative and bigger than BA, unless BO_2 is equal to or greater than $2AO$,

i.e. C lies outside the lens.

(2) If the curvatures are in opposite directions, AC is negative and always less than BA. That is, C always lies inside the lens, and if the thickness BA is very small, C may be taken in the middle point.

(3) If one surface be plane, $AC = O$ or $-BA$. Therefore C lies on the curved surface.

69. It is very necessary to gain a precise idea of what this centre is. Suppose that a ray of light met the first surface at P in such a way that it was refracted in direction PQ. Since at Q the tangent plane is parallel to that at P, the effect on the whole would clearly be the same as if the ray had been refracted through a plate, i.e. it would come out of the lens parallel to its former direction.

We shall, for the future, consider only such lenses as those in (2), and suppose that they are very thin. Then, by what has just been said, any ray that passes through the middle point of the lens is refracted without any deviation.

70. We have hitherto considered lenses only in cases when the luminous point was situate on the axis. But, of course, when we look at any object through a lens there can be but one point on the axis. It is therefore very necessary to know where the images of the other points of the objects are formed. We are able to do this by the use of the centre. As however the investigation involves considerable analysis, and is not necessary to our future work, we will only give the result, which is extremely simple. It is, as usual, only *very approximately* true.

Fig. 51 (i).

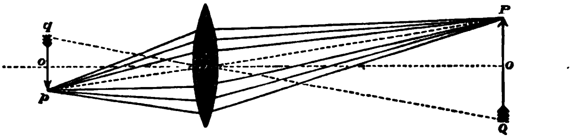
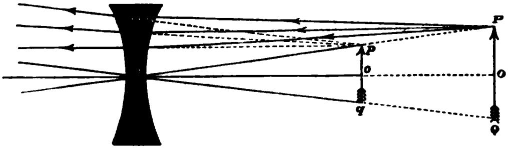


Fig. 51 (ii).



Let QCq be the ray, from Q the luminous point not on the axis, which passes through the centre of the lens.

Then the result is this.

The image of Q may be regarded as formed at q, where the same formula obtains as for direct refraction, viz.

$$\frac{1}{Cq} - \frac{1}{CQ} = \frac{1}{f}.$$

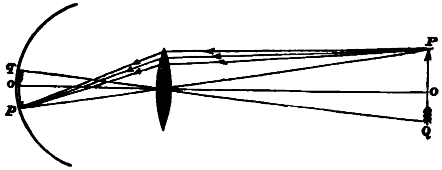
As usual we consider only a very small pencil close about the axis QCq.

The image therefore will appear inverted in fig. 51 (i), and upright in fig. 51 (ii). In both cases it will appear slightly curved, for from the equation $\frac{1}{Cq} - \frac{1}{CQ} = \frac{1}{f}$ we obtain $\frac{Cq}{CQ} = \frac{1}{f + CQ}$, which shews that although Cq increases with CQ, yet the proportion of Cq to CQ decreases. If the incident rays were parallel we have $Cq = f$, and therefore the image is part of a circle whose radius $= f$.

71. We are now in a condition to see how the use of a lens can assist vision; and without at present giving an accurate description of the human eye, we will merely say that it is in effect one double convex lens which forms images, and a retina or network of nerves which receives the images and transmits the impressions which they convey to the brain. The retina is spherical.

If now the object POQ be just at the right distance for most distinct vision, i.e. so that the image of the centre of the object lies upon the retina, it is clear that, all the other circumstances remaining the same, the image of an object further than PQ would be formed in front of the retina, and that of an object nearer than PQ would be formed behind the retina, by (1) in Art. 65. But in some manner not at present determined, the relative circumstances of the lens and retina can be altered at will within certain limits and

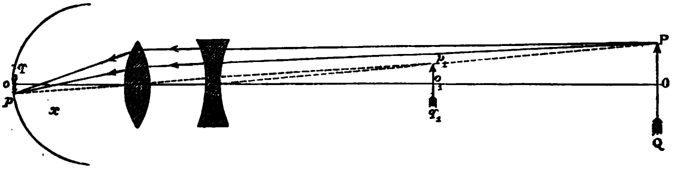
Fig. 52.



with very great rapidity, so that in any case the image is thrown upon the retina.

72. When, in spite of this power of self-adaptation, the image of a distant object is formed in front of the retina as at x in the figure 53, such an eye is said to have *short sight*, which is occasioned by the too great convexity of the lens. It is clear that if by means of another lens we are able to counteract this convexity, we shall rectify this defect, and a little consideration will shew that a double *concave* lens is the best to use, since it performs the required office when held close to the eye, and forms an upright image, while a double *convex* lens, although it would bring the object nearer as required, yet, since it forms its image between the eye and the lens, would have to be held at an inconvenient distance from the eye, and the image would be inverted.

Fig. 53.



But now suppose that the image of a near object is formed behind the retina. Such an eye is said to have *long sight*, and can have this defect corrected by any lens that will throw the image farther back. Such a lens is a convex lens with a very long focal length, as can be seen by fig. iii. Art. 65.

The eye of a fish is of course adapted to see objects in the water, a dense medium. The lens in the eye must therefore be either very dense or nearly spherical so as to bend the rays sufficiently. The latter is the case. Thus, immediately a fish is brought out of the water, it is afflicted with very short sight.

"The long-sighted eye receives greater benefit from a convex lens than a short-sighted eye does from one that is concave. For an object seen through a convex lens is not only magnified, but it is seen brighter by the eye receiving a larger pencil of light from each visible point, on account of the rays entering it less divergent. On the contrary, a concave lens not only diminishes objects, but also renders them darker, because the rays are thus transmitted more diverging to the eye, and consequently the pupil cannot receive so many of these rays as it otherwise would. It is on this account that short-sighted people cannot see remote objects, unless they be very large and bright, so well through a concave lens, as the theory of images formed on the retina may seem to promise. For the principal impediments to the distinct vision of remote objects are their want of light and magnitude, and both these defects are farther increased by concave lenses." (*Encyclopædia Metropolitana.*)

CHAPTER XI.

Telescopes.

73. Roger Bacon, A.D. 1250, has treated, among other Optical Phenomena, upon the apparent magnitudes of objects and spherical foci. He says, "For it is easy to understand, by the canons above mentioned, that the greatest things may appear exceeding small, and contrarily. For we can give such figures to transparent bodies, and dispose them in such order, with respect to the eye and the objects, that the rays shall be bent and refracted towards any place we please; and thus, from an incredible distance, we may read the smallest letter, and may number the smallest particles of dust and sand, by reason of the *greatness of the angle* under which we may see them; and, on the contrary, we may not be able to see the greatest bodies just by us, by reason of the smallness of the angle under which they may appear. *For distance does not affect this kind of vision*, excepting by accident, *but the angle does.*" Thus Bacon really produced effects precisely analogous to those produced by our telescopes and microscopes.

In 1570, in the first English edition of Euclid's Elements, by Sir Henry Billingsley, there are mentioned "perspective glasses," clearly the same as our telescope. In 1591, Thomas Digges of Oxford writes thus: "My father, by his continuall painful practises, assisted with demonstrations mathematicall, was able and sundry times hath, by proportionall glasses, duly situate in convenient angles, not only discovered things farre off, read letters...but also seven miles off declared what hath been doone at that instant in private places."

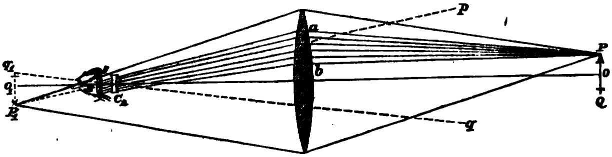
In 1610, so powerful had these glasses been made, Harriot was able to observe the spots in the sun.

Towards the end of the 16th century, a fortunate concurrence led to the invention of dioptric telescopes in Holland. Borelli informs us that the children of Zachariah Jansen, a spectacle-maker of Middle-Burgh, amusing themselves in their father's shop, placed by chance a convex and a concave lens in such a way, that on looking through them at the weathercock of the church, it seemed to them much larger and nearer than usual...This was about 1590. No sooner was this interesting discovery made known...than Galileo, Kepler, Descartes and other philosophers bent the whole force of their genius to the improvement and employment of so useful an apparatus. (*Encycl. Metropol.*)

74. It would be well, before discussing what it really was that these children had found out, to give a slight explanation of the words in italics in Bacon's writing quoted above.

If we hold a small circular piece of paper between us and the sun, we can, by placing it at the proper distance, exactly eclipse the sun: and we say then that the sun and the paper appear the same size. If we did not know the distance of the paper or its size, the eye would at once convey to the brain the idea that the sun and paper were of the same size. In a total eclipse, for instance, of the sun by the moon, this occurs. The moon though much smaller than the sun, appears to cover the sun, that is, appears to be the same size. Thus size is judged by the relative *angles*, not by the relative lengths and breadths. It will be necessary constantly to bear this in mind in what follows.

Fig. 54.



75. Figure 54 will shew how it was that the children of Jansen saw the weathercock larger, by means of a convex (C_1) and a concave (C_2) lens.

Suppose for the moment that C_2 was removed. The rays from P of the weathercock would, after refraction at C_1, form an image at p_1, where PCp_1 is a straight line, and $\frac{1}{C_1p_1} - \frac{1}{C_1P} = \frac{1}{f}$ where the quantities have their proper signs, according to the usual convention. Similarly Q would have its image at q_1. Now let the concave lens C_2 be placed so that p_1q_1 is just in its focus. The rays then which are coming from the object-glass to the focus p_1, and are intercepted by the lens C_2, will on emergence from C_2 form a beam of rays parallel to p_1C_2. But when rays enter the eye in a parallel beam, they appear as if they came from a point a very long way off. Thus, then, these rays appear to come from some point p (i.e. the eye sees an image) a very long way off, in the direction p_1C_2 produced. Similarly with q. So that, finally, the object PQ, by means of the lenses, is seen under an angle pC_2q instead of under the angle PEQ or PC_2Q, as it would be without the lenses.

We have in the last few words assumed that the eye and the eye-lens coincide. In order to gain the central rays of the pencil falling on the eye-lens from each point of the object, it is clear that the eye must be placed close to the lens. Whence it follows that there is no object in having an eye-lens much larger than the pupil of the eye. Thus, by the figure, only a very small portion of the rays falling on the object-glass come to the eye.

We have therefore now shewn how an image is formed *larger* than PQ. But the children saw it also *nearer*, whereas, by our method, it has been placed at an infinite distance. A very slight alteration in the position of the lenses will accomplish this. Suppose we push in our concave lens very slightly, nearer to C_1. The result of this, since an object and image move in the same direction, will be to give the emergent rays a slight divergence from some point p on p_1C_2 produced, not at an infinite distance, as before. The *size* of the final image pq remains the same as before, since it is measured by the angle under which it is seen, and that is not altered, except by the very slight movement of C_2.

For objects so distant as the moon, for example, or indeed for objects much closer, at the distance of a few hundred yards, the rays incident on the object-glass may be considered parallel, and p_1q_1 is therefore in the principal focus. Therefore C_1C_2 = difference of focal lengths of the two lenses.

76. This combination of lenses is called Galileo's telescope, from the following facts. "Happening to be in Venice in the month of May 1609, he learned in that city that a Belgian had invented a perspective instrument, by means of which distant objects appeared nearer and larger.

This report...excited an intense interest in his mind, and upon his return to Padua he began to ponder upon the probable mode by which such an effect could be produced. He discovered upon the very night of his arrival, that the enlargement of the object depended upon the doctrine of refraction, and on the following day he made his first attempt to execute an instrument upon this principle. Having procured a leaden tube, he fitted in one of its extremities a plano-convex lens, and in the other a plano-concave one, and, having applied his eye to the concave lens, he was delighted to perceive objects pretty much magnified. They appeared to be three times nearer and nine times larger....He immediately gave intimation of his success to his friends in Venice, with whom he had been conversing on the previous day. Having succeeded a few days later in making a better instrument, he joyfully proceeded to Venice, taking it with him....Galileo's object now was to construct a telescope of superior magnifying power. This he found to be a task of very difficult accomplishment, for the art of grinding and polishing lenses was then in its infancy....Soon afterwards he made another instrument which magnified 60 times in surface, and finally, sparing neither application nor expense, he succeeded in executing an instrument of such excellence as to represent objects almost 1000 times larger, and above 30 times nearer than they appeared to be by the natural power of the eye." (Grant's *Hist. of Astron.* Vol I. p. 520.)

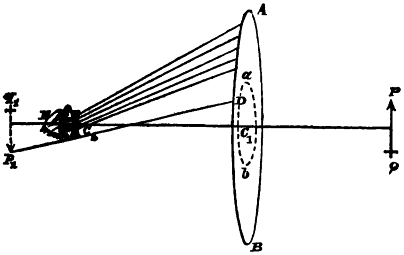
Thus Galileo constructed this telescope *by theory*, by thinking *how* it could be done, whereas all former constructions had been entirely experimental.

77. There are two great disadvantages attending this construction.

(i) The image q_1p_1 is *virtual.* The disadvantage of this will be best seen when we discuss the Astronomical Telescope.

(ii) The small number of rays falling on the object-glass which come to the eye lens. This will be best seen by tracing the rays backwards from the eye to the part of the object-glass from which they have come.

Fig. 55.



Suppose then that the eye of perfect sight receives a beam of rays which have originally come from *P.* They are, of course, after emergence from the eye-lens, parallel to p_1C_2. Before entering the eye-lens these rays have formed a converging beam from the object-glass. Thus if the eye is to receive such a beam as we have drawn, the object-glass must be of the size *AB*, much larger than it is possible to make it. Even then, no rays from *P* which fall on the object-glass below *D* would come to the eye. Thus with an object-glass as *ab*, hardly any rays, starting from *P*, would come to the eye, so that scarcely any image of *P* would be seen. In the Astronomical Telescope, as will be seen, usually all the rays which fall on the object-glass fall also on the eye-lens and are of use.

78. The proportion of the angle which the image seen by the eye subtends at the eye to the angle which the object would subtend at the eye, without the lenses, is called the "*magnifying power*" of the telescope.

Thus in fig. 53 "magnifying power" $= \frac{pEq}{PEQ}$.

Since the eye is close to the lens, we may take C_2 instead of E.

Thus

$$\text{“magnifying power”} = \frac{pC_2q}{PC_2Q} = \frac{p_1C_2q_1}{PC_2Q}$$

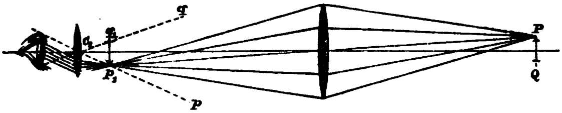
$$= \frac{PC_2}{p_1C_2} \text{ very nearly} = \frac{\text{focal length of O. G.}}{\text{focal length of E. G.}}.$$

Galileo points out a simple method of determining by experiment the magnifying power of the instrument. "Place," says he, "upon a wall at a certain distance two disks, one of which you will observe with the telescope, and one with the naked eye. If the disk seen through the telescope appear equal to the external one, the magnifying power is in the proportion of the two disks. If they do not appear equal, enlarge or diminish the external disk until this result takes place, and then the magnifying power will be as mentioned, in the proportion of the two disks" (Grant's *Hist. of Astronomy*).

79. Two years after Galileo's discovery of this telescope, Kepler first explained the construction of a telescope formed by the combination of two convex lenses, 1611. He did not, however, manufacture one himself. In 1617, Scheiner made observations in the presence of the Archduke Maximilian with such an instrument.

In this case the eye-lens is placed so that the image p_1q_1 is between it and the object-glass, and is in its focus. The rays then which have come from P, after emergence from the eye-lens, come out in a beam of rays parallel to p_1C_2. That is, the eye behind C_2 sees an image p of the point P at an infinite distance in the direction C_2p_1 produced. Similarly with Q.

Fig. 56.



As before, the rays from P to the object-glass may be supposed parallel, so that p_1q_1 is in the focus of C_1. Therefore $C_1C_2 =$ *sum* of focal lengths. By pushing our eye-lens slightly nearer to p_1q_1, we could give the emergent rays a slight divergence, as shewn in the Galilean telescope, and thus make them appear to come from a point p *not* at an infinite distance.

80. Now we have to remark that here we have made, for convenience of drawing, our eye-lens much larger relatively to the object-glass than it really is in practice. This was done for the sake of taking in the whole pencil. But if we have a very small eye-piece in the same position we should lose some of the rays from p_1 for instance. This we could obviate partly by taking an object-glass c_2 with a greater focal distance, since then the rays from it would converge to and diverge from p_1 more slowly, and so the eye-piece would take more of them in. And the greater the focal length, the more would this be the case. ("Thus Huyghens states that he and his brother made object-glasses

of 170 and 210 feet focal length. These glasses were used without tubes. He presented one, having a focal length of 123 feet, to the Royal Society of London. While he was engaged in these labours the art of executing telescopes was attaining a high degree of excellence in Italy, but they were constructed chiefly in France and England. Auzont made one of 300 feet focal length.") If, again, we had an eye-piece with a very small focal distance, we should thus catch more rays, since the eye-piece would be closer to the image, but we should in that way have a greater magnifying power than is convenient.

We saw in Art. 76, that Galileo fitted his lenses at the ends of a tube. The use of this tube was to shut out the interference of all light that was not actually being used, *i.e.* all except the rays from the object on the object-glass. Thus, what a refracting telescope does is this. It collects a great many rays from a point of the object and sends them to the eye in a dense small beam of parallel rays (whereas the eye without the lenses would only receive the rays that fell on its pupil), thus giving to the eye an image of the point at an infinite distance. If the eye is not long-sighted enough for parallel beams, by shoving the eye-piece slightly in, we can, as said in Arts. 75, 79, give the rays the required amount of divergence.

81. Comparing the Galilean and astronomical telescopes we notice the following differences.

(1) In the Galilean the distance between the lenses equals the *difference*, in the astronomical it equals the *sum*, of the focal lengths.

The Galilean construction is therefore useful for opera-glasses, &c. which require to be short.

(2) In the Galilean only a small portion of the rays which fall on the object-glass are of use. In the astronomical usually nearly all are of use. See figs. 55, 56.

Thus, in fig. 55, if a piece of paper were placed in front of the portion *ab* of the object-glass, no image of *P* would be seen at all, since it would cover the only part of the object-glass by which *P* is seen. On the other hand, if any portion of the object-glass of the astronomical in (56) were covered, the only effect would be to dim the brightness of the whole.

(3) In the Galilean the image is *erect:* in the astronomical *inverted.*

(4) In the Galilean the image is *virtual*, in the astronomical it is *real.* This advantage of the astronomical telescope and (2) quite put out of sight the comparatively small advantages (1) and (3) possessed by the Galilean.

"The first person who appears to have discovered the peculiar value of the telescope recommended by Kepler, and who applied it to the purposes of astronomical investigation, was Gascoigne. He did not fail to perceive that it possessed an immense advantage over the Galilean telescope in affording a situation within the tube where any object being placed might be viewed as distinctly as a celestial object. This suggested to his inventive mind the use of telescopic sights in astronomical observations, and the application of the micrometer to the telescope." If, for instance, a series of wires were placed inside the telescope at the focus where p_1q_1 is formed, the eye would see both wires and image, and the relative breadths, for instance, of two planets might be found by noticing how many wires they respectively covered; or a ring might be placed there to limit the amount of the image that the eye was to see.

What we have called 'wires' in telescopes are in reality threads of a spider's web.

(5) In the Galilean the eye should be placed close to the lens. In the astronomical it should be at *E* when the central rays of the pencils from the extreme points of the object cut the axis, since otherwise it would lose a great many of the rays.

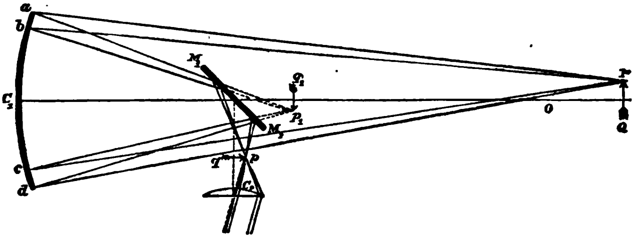
82. The great length necessary in these telescopes made it desirable to discover some new construction. In a letter written to Descartes in 1639 Mersenne suggested the idea of a reflecting telescope. Descartes offered several objections to the proposal, and it was not carried into effect. It is impossible to ascertain whether Gregory was indebted to any previous writer for the hint of the reflecting telescope or not: but, at any rate, he was the first person who gave an explanation of it.

The first person who actually executed a reflecting telescope was Newton. But the difficulties in his way were very great, since every irregularity in a reflecting surface would make the rays deviate five or six times more out of their due course than in a refracting telescope. While his mind was occupied with these reflexions, he was compelled by the plague to leave Cambridge, and more than two years elapsed before he again considered the subject. Accordingly, having succeeded, after much trouble, in discovering a substance adapted for the formation of his larger mirror, and having also devised a delicate mode of polishing the surface, he actually executed a telescope upon this principle. The original idea had been to make a parabolic mirror, which, as we saw (Art. 15), reflects parallel rays accurately to the focus. Newton however wisely abstained from attempting to make it truly parabolic, on account of the extreme difficulty of the operation, contenting himself with giving it the figure of a segment of a sphere. With this instrument he could see the satellites of Jupiter and the horns of Venus. Encouraged by this success, he made a second telescope, still better than the first. The focal length of the speculum was $6\frac{1}{3}$ inches, and that of the eye-glass (which was a plano-convex lens—the flat side turned towards the eye) was $\frac{1}{6}$ of an inch. The magnifying power was therefore 38. See Art. 78.

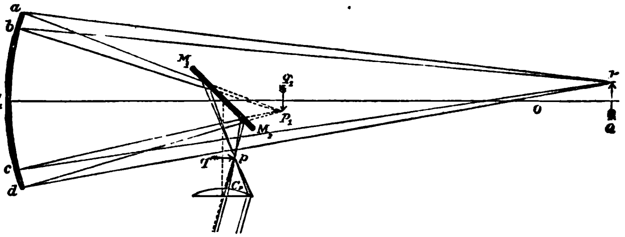
In December, 1671, Newton sent this telescope to London, to the Royal Society, from whom it received unanimous commendation. A description of it was sent to Huyghens in Paris, by whom it was communicated to the Academy of Sciences, and in consequence it was soon generally known on the continent. (Grant's *Hist.* p. 528.)

83. We will now give a sketch of the telescope devised by Newton.

Fig. 57.



C_1 is the mirror which Newton made in the shape of the segment of a sphere instead of that of a parabola. We will suppose that PQ is so far off, that the rays Pa, Pb are to be considered parallel. Then an image of PQ would be formed at p_1q_1 where C_1p_1 and C_1q_1 equal $\frac{1}{2}$ radius of C_1 (Art. 22). PQ should, of course, be much further off, so that p_1OP, q_1OQ may be straight lines. But Newton placed a plane mirror M_1M_2 inclined at an angle of 45° to the axis, the effect of which is that the rays coming to p_1 are reflected off to p, and those from q_1 to q, where the lengths of the reflected rays just equal what they would be if there were no mirror (Art. 6). And $pq = p_1q_1$ and pq is at right angles to p_1q_1. The plano-convex lens C_2 is placed with its plane parallel to the axis so that pq is exactly in its focus (supposing the sight to be perfect sight), whence the rays which fall from p emerge in a parallel beam, and the image is seen at an infinite distance in the direction pC_2.



84. An object must be to make M_1M_2 as small as possible, since it intercepts a great many rays and so prevents them from coming to C_1. If we join q_1 to the top of the mirror and p_1 to the bottom, those lines form the boundary of the reflected rays from PQ and therefore of the plane mirror. If we make it larger, we shall cut off some rays from PQ from reaching C_1, and gain no recompensing advantage. If we make it smaller, more rays will fall on C_1, but the extreme rays will not come to M_1M_2.

85. From the figure we see that the object is seen at right angles to its proper direction. Two other kinds of reflecting telescopes were invented, one before Newton's, by Gregory (not however manufactured till afterwards, by

Hooke, 1674), and the other by Cassegrain, a Frenchman, in 1672. The advantages of these were that the image was viewed directly instead of at right angles.

In 1723 Hadley presented to the Royal Society a reflecting telescope of the Newtonian form, the diameter of the large mirror being six inches, and its focal length five feet three inches. When compared with the refracting telescope of 123 feet focal length, presented to the Society by Huyghens, it was found that it bore as high a magnifying power as that instrument, and exhibited objects equally distinct, though not so bright.

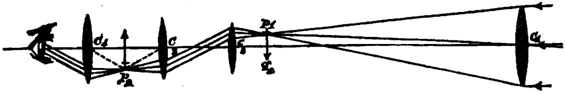
As noticed in other telescopes there is great advantage in having a mirror with a long focal distance, since the small plane mirror will then catch so many more of the rays reflected off the big mirror.

"The telescopes which Herschel employed in the early part of his astronomical observations were all of the Newtonian construction. In 1786, however, he laid aside the small mirror, and instead gave a slight inclination to the large mirror, so as to throw the image a little to one side of the tube, and thus make it possible to view it directly with an eye-glass. By this contrivance the light usually absorbed by the small mirror was saved, and the illumination of the image increased in a corresponding degree. In such a telescope it is obvious that the observer looks at the image with his back turned to the object. In 1789 Herschel surpassed all his former efforts by the completion of a telescope forty feet focal length and four feet aperture. It is impossible, within a moderate compass, to give even a simple enumeration of the multitude of brilliant discoveries in celestial physics which rewarded the labours of this great astronomer."

The advantages of these reflectors obviously consist in their being much shorter than refractors of equal power, and in the facility with which different eye-pieces may be applied to the same speculum. Among their disadvantages are the difficulty of directing them to the objects, but particularly the extraordinary precautions that are necessary as well in the application of them as in their construction: they are also very expensive, in consequence of the metal speculums, and are easily put out of order.

86. We stated in Art. 81 that a slight advantage of the Galilean telescope over the astronomical was to be found in the fact that the image seen in the former was erect, while in the latter it was inverted. We give a sketch of an eyepiece which might be used to counteract this defect in the Astronomical Telescope, not from any practical knowledge that might be gained by it, since it has long been superseded by improvements, but because it gives an useful exercise in tracing the course of a small pencil.

Fig. 58.



p_1 is just in the focus of C_2, therefore the rays will emerge parallel to p_1C_2.

After refraction at C_3 the parallel rays will be bent to the focus p_2 of C_3.

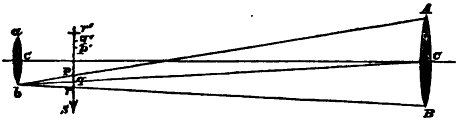
If p_2 is just in the focus of C_4, the rays will emerge parallel to C_4p_2.

Thus, finally, the object p is seen in direction C_4p_2 instead of C_2p_1.

87. In Art. 81 (4) it was mentioned as one of the great advantages which the astronomical telescope possessed

over the Galilean, that from the fact of the image being *real* instead of virtual, it was possible to limit the amount which the eye wished to see, by placing a ring in the focus where the image was formed. We will give some further explanation of this.

Fig. 59.



Draw straight lines from *b* the bottom of the eye-lens to *A*, *C* and *B*, the top, centre, and bottom of the object-glass, and let them cut the image in the points *p*, *q*, *r*.

Suppose now that a ray came from *A* to any point of the image below *p*. It will pass *below* the eye-lens, and the eye will not get the benefit of it. Thus the image will be slightly dimmer just below *p*. And as we look lower and lower at the image this dimness will increase, since more and more rays will pass underneath the eye-lens. And when we come to *q*, just half the rays are of use, viz. those falling on the lower half of the object-glass. Then *c* is said to be seen by half pencils. Below *q* the image will be seen by less than half pencils, less and less, and *r* is the last point that will be seen at all, since of all the rays converging from *ACB* to any point *s* below *r*, not one comes to the eye-lens. Similarly, no point above *p'* will be seen quite clearly. This indistinct piece in the image is called the "*Ragged Edge:*" it is found convenient to cut it off from sight, which is done by the ring in the focus, the aperture of which equals *pp'*, so as just to allow the whole of the distinct portion of the image to be seen. This ring is called the "*Stop.*"

This could not be done in the Galilean telescope, since the image does not really exist.

The size of this "Ragged Edge" can be found thus,

$$\frac{pr}{AB} = \frac{bq}{bc},$$

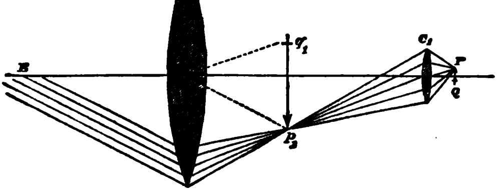
$$\text{or } \frac{\text{Ragged Edge}}{\text{diam. of object-glass}} = \frac{\text{focal length of eye-glass}}{\text{f. l. of eye-glass} + \text{f. l. of obj.-glass}}.$$

88. There is one more instrument which we will describe, as it differs in its object from the telescope already discussed, although in construction it is very similar, viz. the *microscope*. We shall give its construction in its simplest form.

The object of the instruments already treated on was to magnify and view clearly very distant objects; they were therefore called "*Telescopes*." The rays on the object-glass might then be regarded as parallel, and after passing through it, converged to the principal focus of the object-glass.

The object of the present instrument is to magnify and view clearly very minute objects at a very small distance: so that the rays can be no longer regarded as parallel on the object-glass. The manner in which this is done can be seen by the following figure.

Fig. 60.



C_1 is a convex lens with a very small focal length. PQ is a minute object placed slightly further from C_1 than its focus. If PQ were exactly in its focus the emergent rays would be parallel to PC_1. Therefore since object and image move in same direction, the rays from P will converge to a point p_1 on PC_1 produced, where

$$\frac{1}{C_1p_1} - \frac{1}{C_1P} = \frac{1}{f}.$$

These rays are then viewed as in the Astronomical Telescope by another convex lens placed so that p_1 is in its focus. The rays then emerge parallel to p_1C_2. The image therefore is seen under the angle $p_1C_2q_1$, whereas without the lenses it would be seen under the angle PEQ.

The smaller the focal length of C_1, as also of C_2, the greater would be the angle $p_1C_2q_1$, *i. e.* the greater would be the magnifying power. Object-glasses are now made for microscopes with focal length of not more than $\frac{1}{50}$th inch. From the fig. it is plain that the eye-lens must be considerably larger than the object-glass.

To explain the fact that a star seen indistinctly through a telescope on account of the brightness of the night, can be made clearly visible by employing a higher magnifying power, while a planet cannot. If a faint star be viewed on a bright evening, it may be invisible on account of the brightness of the field of view.' If now we increase the magnifying power, we do not increase the total quantity of light that enters the telescope, because that only depends upon the size of the object-glass. But by increasing the magnifying power we spread the same light over a larger area, and therefore the field of view appears fainter. If the star be a planet it will have a sensible disk, which will be magnified in the same proportion as the field of view, and the relative brightness will therefore remain unaltered. But in the case of a star there will be *no* sensible disk, and the brightness of the star will therefore be unaltered, that is, will appear relatively brighter than the field of view.

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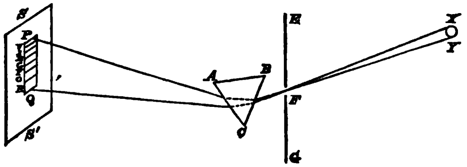
CHAPTER XII.

Chromatic Dispersion.

89. We have, up to the present point, assumed that whatever happens to a ray of light, whether it be reflected or refracted, its *quality* remains always the same; that it is like a ball, which, after being thrown against a wall, is exactly the same ball as before it struck the wall. It was not until many attempts had been made to explain the Rainbow, that Newton stated, in 1675, the theorems that "Lights which differ in colour differ in refrangibility," and "The light of the sun consists of rays differently refrangible." The latter of these is the one with which we shall be chiefly concerned, but Newton's first experiment to prove the first of them is so simple and instructive, that we give it.

He took a black oblong stiff paper terminated by parallel sides, and painted the upper half red and the lower half blue, and viewed it through a prism of a refracting angle of 60° held parallel to the sides of the paper and the cross line. He then found that if the refracting angle of the prism were turned upwards, so that the paper might seem to be lifted up by the refraction, the blue half was lifted higher than the red half: and if downwards, the blue half was carried lower than the red half: which shewed that in both cases the light from the blue half of the paper suffered a greater refraction at the prism than that from the red half. We will now state the experiment by which Newton proved the second of the theorems above, viz. that "Light of the sun consists of rays differently refrangible."

Fig. 61.



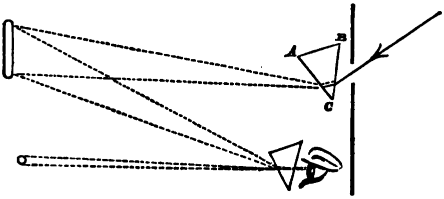
He allowed the sun to shine through a round hole *F* in a shutter, about $\frac{1}{3}$ of an inch in breadth. He then placed a prism *ACB* behind the hole, so that for the very small pencil from *F* falling upon it it should be in the position of minimum deviation for convenience, which position he settled in the way given in (Art. 56). Some distance behind the prism he placed a screen *SS'* to receive the refracted rays. The image *PQ* formed was oblong and not oval, but terminated with two rectilineal and parallel sides by which it was bounded pretty distinctly, while its ends were confused and indistinct, the light there decaying and vanishing by degrees. The *breadth* of this image answered to the Sun's diameter. This elongated stripe *PQ*, the height of which is about five times its breadth, is found to be of different colours, these colours being ranged in the following order: violet, indigo, blue, green, yellow, orange, red.

90. This experiment then shews that a ray of sunlight is not one colour by itself, but is made up of different colours, between which the eye does not distinguish, until they are detected for it by the new medium, which does not act on one colour in the same way as on another.

It is easy to see why the colours in this stripe, which is called the solar "*spectrum*," are not pure, *i.e.* appear to run into one another. Each separate ray of the small pencil from *F* has its own spectrum: thus the yellow, orange, and red of the spectrum of the uppermost ray would come upon the blue, green, and yellow of the middle ray, and upon the violet, indigo and blue of the lower ray, and thus no colours would be perfectly pure, except the extreme edges of the violet and red.

91. Newton also shewed that all the various portions of this coloured band, when again brought together, produce upon the eye the effect of white light.

Fig. 62.

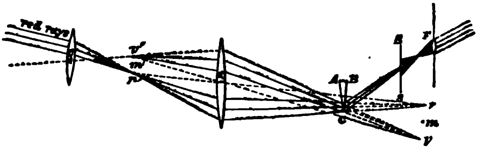


This experiment he performed by simply allowing the light passing through the round hole *F* to fall on *ABC*, producing the solar spectrum, and then, on looking at this coloured band by another prism (*abc*) placed in the same direction, instead of seeing a coloured band, he observed a spot of white light. (Roscoe.)

92. We stated in Art. 90, that the spectrum was not quite pure. But in order to examine the spectrum usefully, it is necessary to have it quite pure. This object we can attain by the following means.

(i) Let a small convex lens *F* of small focal length be placed in the hole in the shutter. An image of the Sun will be formed in its principal focus, and since the focal length is very small the image will be very small, so small indeed that we may regard it as a mere luminous point.

Fig. 63.



(ii) Let a very small pencil from this image of the sun fall upon the prism *ACB*, which is placed in the position of minimum deviation for mean rays, *i.e.* for rays of a mean amount of refrangibility, the remainder of the rays from the image being cut off by *SS'*. This prism refracts the violet rays most, and the red rays least. We will consider only those two colours for simplicity. Since our prism is placed in the position of minimum deviation, the red rays will appear very nearly to diverge after emergence from a point *r*, and similarly for violet (Art. 57).

(iii) Let the rays now be received on the convex lens C_1.

The red rays will then converge to r', where rC_1r' is a straight line, and $\frac{1}{C_1r'} - \frac{1}{C_1r} = (\mu_r - 1)\left(\frac{1}{r} - \frac{1}{s}\right)$: similarly for the violet, μ_r being the index of refraction for red rays.

The spectrum $r'v'$ is pure, but is too small to be observed, since clearly it is hardly more than mere *points* of colour, therefore

(iv) Lastly, let them be received upon another convex lens C_2, so placed that m', the point to which the mean rays converge, is in the principal focus. The rays from r' will then emerge parallel to $r'C_2$, those from v' parallel to $v'C_2$, and we can by placing our eye at the proper place behind the lens C_2 view the image of the spectrum $r'v'$ directly. This is obviously using the simple astronomical telescope to view the object rv.

93. The great practical difficulty in performing this experiment when tried with sunlight, consists in the fact that as the sun moves, the light does not fall in the same direction on F, and its image therefore moves. To obviate this we must have, outside the shutter, an instrument called a "heliostat." This is a mirror revolving about an axis parallel to the Earth's axis with one-half the sun's angular velocity, so that it shall always reflect the sun's rays in a fixed direction, viz. on to the lens.

94. It can be shewn that the spectrum thus found cannot be any further broken up. If we take for instance only the red rays in the last figure, after emergence at C_2, and let them fall again on a prism, their colour is unchanged at emergence. Thus, finally, the foregoing experiments shew us that ordinary light is made up of a number of rays of different refractive indices varying between close limits.

95. In Art. 91, we shewed how Newton proved that all the various portions of this coloured band, when again brought together, produce upon the eye the effect of white light. The same thing can be proved, or rather illustrated, by a very easy experiment, which depends upon the fact

that light remains a short time on the eye. This fact can perhaps be shewn most simply thus. Place a white bead at the end of a string, and whirl it round. At first the eye is able to follow the bead round and round, but when it attains a certain pace, the eye sees only a white ring. This then shews that the light which came to the eye when the bead was in any fixed point of its orbit has remained on the eye during the time that the bead takes to make one revolution. This time is somewhere about $\frac{1}{15}$th of a second.

If now we take a disk of cardboard, and paint it in sectors with the colours of the spectrum in their proper proportion, we shall, if we place it on a top revolving 15 times per second, gain the same effect as if the colours were mixed together in the same proportion, that is, we shall see a *white* disk.

Now the reason why all these different coloured lights, when they reach the eye, produce the effect of white light, is a physiological question which we cannot explain. We find that not only do all the colours of the spectrum, when they are brought upon the eye, produce the effect of white light, but that several mixtures of only a few out of all these different colours have the same power of producing upon the eye the effect of white light: for instance, violet and greenish yellow; indigo and yellow; blue and orange; greenish blue and red.

96. There is a curious analogy between the human eye and ear which this spectrum brings out. It is well known that there are certain sounds which, and higher than which, many ears cannot hear at all, such as a grasshopper's chirp. The sensation of sound is produced by the vibrations of air acting on the nerves of the ear, each set of nerves being acted upon by one sound. If then the nerves

upon which such a sound as that mentioned would act are wanting, the ear would not hear that sound.

Now in the same way it is known that the refracted rays do not end with the visible part of the spectrum. This is found from the fact that outside the spectrum the heating power can be felt although there is no light. Indeed the maximum heating effect is obtained at a point outside the red portion of the spectrum. The blue and violet portion of the spectrum is that least endowed with heating power, but this portion and the invisible portion extending to some distance beyond the violet end are of very great service from the fact that they produce chemical action, such as the decomposition and blackening of silver salts, upon which the art of photography is based.

97. The angle between any one of the coloured rays and the white ray produced is called the "*deviation*" of that coloured ray.

The difference between the deviations of two colours is called their "*dispersion*," and the difference between the deviations of the extreme colours is called the "*dispersion of the pencil.*"

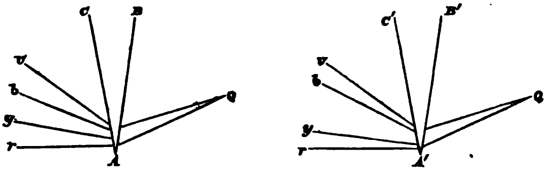
In treating on prisms we shewed (Art. 55) how to find the value of the refractive index μ for any medium. Now if we find in this way the values of μ for all the colours of the spectrum in several different media, we discover the fact that the ratio of the dispersion of any two colours to the dispersion of the extreme colours of the spectrum *is not constant* when different media are used.

Thus, if ABC, $A'B'C'$ be two different media, and the angle between the violet rays and red rays, v and r, is the same in both, it is found that the angle between b and y, the blue and yellow rays, is not the same.

This is called the "*Irrationality of dispersion.*"

98. Hence, if two prisms can be made so as to give the same *total* dispersion, and if they be placed so as to refract in opposite directions; in the emergent beam the extreme red and violet will be united, but not the intermediate colours completely.

Fig. 65.



Since our prism is supposed placed in the position of minimum deviation, if μ_x, μ_y be the indices for any two colours and D_x, D_y their deviations, the angle of the prism being small, then $D_x = (\mu_x - 1)A, D_y = (\mu_y - 1)A$; $\therefore D_x - D_y = (\mu_x - \mu_y)A$, Art. 53; and therefore the dispersion varies as A, but besides this it varies as $\mu_x - \mu_y$, that is, (i) "the *same* substance disperses *different* colours differently." Again, if we take several prisms with the same refractive angle, but made of different media, the dispersion for two fixed colours is not always the same. Thus (ii) "*different* substances disperse the *same* ray differently."

99. It is found also that the dispersion of any two colours is not proportional to the deviation of the axis of the pencil. This is clearly a most fortunate circumstance. If it *were* proportional, another way of saying it would be

that any combination of prisms or lenses which destroyed dispersion must destroy deviation too. Now the fact that a lens or a series of lenses magnifies an object arises simply from the fact that the direction of each ray suffers deviation, and the greater this deviation, the more is the object magnified. Thus then we should, by destroying dispersion, gain a perfectly colourless image, but in no degree magnified.

Since however dispersion is *not* proportional to deviation, media can be found which produce *the* SAME DISPERSION *in opposite directions of a given colour relatively to another*, but DIFFERENT DEVIATIONS in opposite directions in the axis of the pencil. If then a pencil be refracted through these media the two colours in question could be united, while the axis of the pencil suffers a deviation equal to the difference of the deviations which these media would produce separately: but, as we said, the other colours would not be quite united, though nearly. (Parkinson's *Optics*, Art. 173.)

100. Let A and A' be the refracting angles of two prisms of different media, and let μ and μ' be the indices in them for one particular colour. The deviations which the prisms would separately produce for this colour are $(\mu - 1)A$ and $(\mu' - 1)A'$, when the angles A, A' are small, and total deviation $= (\mu - 1)A + (\mu' - 1)A'$. If $\mu + x$ and $\mu' + x'$ be the indices for another colour, the total deviation for this colour would be $(\mu + x - 1)A + (\mu' + x' - 1)A'$. If these deviations be the same and opposite, i.e. if the colours are to be united at emergence from the second prism, then $xA + x'A' = 0$, or $\frac{A}{A'} = -\frac{x'}{x}$, (i).

We have thus obtained one relation between A and A'. If now we have another given, for instance that the whole deviation in a given colour is to be a given amount, then $(\mu - 1)A + (\mu' - 1)A' =$ given quantity, (ii), and from equations (i) and (ii) we can find A and A' and thus make our prisms such that the colours considered may be united and the required deviation in a given colour produced. The *minus* sign in (i) shews that the edges of the prisms are to be turned in different directions.

CHAPTER XIII.

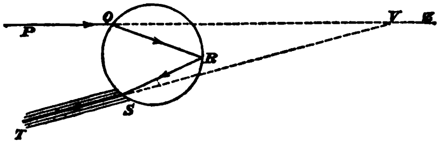
THE RAINBOW.

101. **Early** attempts were made to account for the colours of the rainbow, and various other phenomena in which colours are seen to arise from transient and unsubstantial combinations of media. Thus Aristotle explains the colours of the rainbow by supposing that it is light seen through a dark medium: "Now," says he, "the bright seen through the dark appears red, as, for instance, the fire of green wood seen through the smoke, and the sun through mist. Also, the weaker is the light, or the visual power, the nearer the colour approaches to the black; becoming first red, then green, then purple. But the vision is strongest in the outer circle, because the periphery is greater; thus we shall have a gradation from red, through green, to purple, in passing from the outer to the inner circle." The same doctrine is found in the work of De Dominis. According to him light is white; but if we mix with the light something dark, the colours arise—first red, then green, then blue or violet. He applies this to explain the colours of the rainbow, by means of the consideration that, of the rays which come to the eye from the globes of water, some go through a larger thickness of the globe than others, whence he obtains the gradation of colours.

Descartes came far nearer the true philosophy of the iridal colours. He found that a similar series of colours were produced by refraction of light bounded by shade, through a prism. "At first," he says, "I doubted whether the iridal colours were produced in the same way as those in the prism; but at last, taking my pen and carefully calculating the course of the rays which fall on each part of the drop, *I found that many more come out at an angle of forty-one degrees, than either at a greater or less angle.* So that there is a bright bow terminated by a shade; and hence the colours are the same as those produced through a prism."

102. At length, in 1672, Newton gave the true explanation of the facts. His discovery consisted in teaching distinctly that the law of refraction was to be applied, not to the beam of light in general, but to the colours in particular. The whole explanation of the rainbow turns on this fact and that of "*minimum deviation.*" It was shewn that if a pencil of rays fall upon the face of a prism, that ray which after refraction at the first surface passed through the prism symmetrically with respect to the sides of the prism, was least refracted, and that, of the other rays, those which came out nearest to this ray came out closest together; so that, if an eye received the refracted rays, it would see most light in the direction of minimum deviation (Art. 62). Now the same thing will happen in the case of a sphere instead of a prism.

Fig. 66.

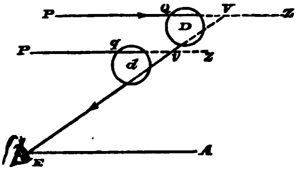


Suppose *PQRS* a ray falling on a sphere: it is refracted at *Q* and falls on the further surface at *R*; part of this ray is refracted at *R* and never meets the eye; but the rest is reflected at *R*, falls on the lower surface at *S*, and is then, for a great part, refracted to *T*.

If now we suppose a great number of rays to fall parallel on the upper side of the sphere, they will come out at the lower side, but more will come out very nearly in the direction of the ray which has minimum deviation than in any other direction. If *PQRST* be this ray, the emergent rays close to *ST* will form a very small pencil of nearly parallel rays, where the angle *TVZ* or deviation = about 139°. An eye therefore that was placed in a position to receive the rays about *TS* would see much stronger light from this drop than an eye elsewhere. We are now in a condition to explain the rainbow generally. We have hitherto in this Art. supposed that the light is unaltered by being refracted: but, on entering at *Q*, it is split up into all the colours of the spectrum.

103. (1) Consider light to be of only one colour, say *red*. We have then a beam of parallel red rays from the sun falling on the drop *D*. The emergent beam will consist of two classes of light, *chief* light and *minor* light, the chief light being that which comes out in the position of minimum deviation, and minor light all the rest.

Fig. 67.



Let *PQ* be the red ray of minimum deviation: let it, after emergence, fall upon the eye in the direction *SE*.

Through *E* draw *EA* parallel to the incident ray *PQ*. Then all the spherical drops of rain which lie on the cone whose vertex is *E* and semi-vertical angle is *AED* send light in the same state to the eye—i.e. send red light in the direction of minimum deviation.

This is apparent by symmetry. There is no reason, for instance, why the light from one drop *D* on that cone should be in the least different from that from any other drop *d* on the same cone.

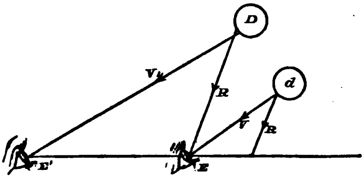
Now recollecting that *VE* is the ray of minimum deviation, and recollecting that the meaning of that in this case is that rays which fall either higher or lower on the drop than *PQ* or *Pq* make on emergence a larger angle *EVZ*, a little consideration will shew that any drop within that cone will send *minor* red light to the eye. But any drop without the cone will send *no light at all.*

The appearance to the eye would therefore be that of a bright red arch, fading gradually away towards the inside of the arch, but sharply defined on the outside.

(2) Now take lights of all the colours of the spectrum. What has just been said applies equally to each of them; and, since each light has its own direction of minimum deviation, we shall have a concentric series of arches of different colours, *EA* being their common axis. Since violet is more refracted than red, the minimum deviation of violet is greater than that of red; whence it follows that the red bow is above the violet bow. The other colours would be arranged in their proper order between. The following figure will, perhaps, help to make this point clear.

If E be an eye and D a drop, such that DE is the position of minimum deviation for the red rays, then E will receive the red rays from D, but not the violet.

Fig. 68.



On the other hand it will receive the violet rays from *d*, but not the red rays.

The red bow is therefore seen in the direction ED, while the violet bow is seen in the direction Ed. That is, the red bow is the outside one.

The *chief* red light is at the red arch, but minor red light, as we said, is within the arch, and so for all the colours. That is, the red part of the bow is the only part that is seen quite pure. Also we noticed that close to the red bow, on the outside of it, there is *no* light; the red bow is therefore the most visible, by contrast.

This rainbow, formed by two refractions and one reflexion, is called the "*Primary*" rainbow.

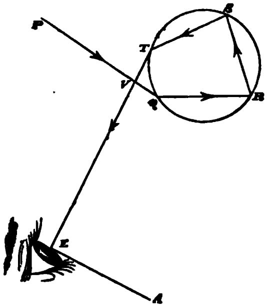
104. The space above the primary bow appears for some distance, as we said in the last Article, darker than the rest. Beyond this space appears a broader but fainter rainbow, the colours of which are in the reverse order to those in the primary. The origin of this "*Secondary*" bow can be explained by pencils which come to the eye after having suffered two internal reflexions at the raindrop. (Parkinson's *Optics*, Art. 267.)

At every reflexion and in a less degree at every refraction the pencil loses somewhat of strength. This will account for the secondary bow being fainter than the primary bow, since one more reflexion takes place in it than in the primary bow.

The following figure will serve to shew the course of the rays forming the "*secondary*" rainbow.

The ray *PQ* is refracted at *Q*, reflected at *R*, reflected at *S*, refracted at *T*, losing some of its brilliancy at each change. Thus an eye at *E* sees a faint illumination in the direction *ET*.

Fig. 69.



The *deviation*, being the angle through which the ray has been bent from its original direction before it meets the eye, is the angle $360^\circ - QVE$. This angle will therefore be greater for violet rays than for red rays, and by reasoning precisely similar to that used in the last Article, the reader can see that the violet arch will in this case be the highest, and the red the lowest; also that the *minor* light of each arc is *above* the arc. Thus the space between the two bows is much darker than that below the primary bow or above the secondary bow.

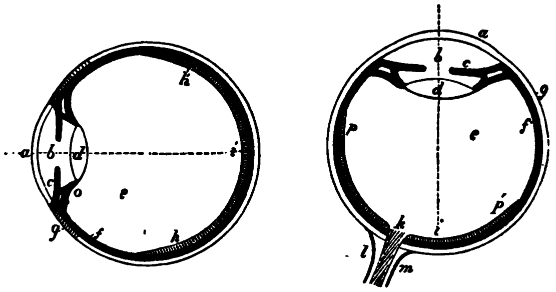
CHAPTER XIV.

THE HUMAN EYE.

105. IN the chapter on lenses we said that for the purposes which we had then in view the eye might be regarded as a double convex lens, forming images of external objects on the retina, which is placed at the back of the eye, and the nerves at the back of which convey impressions to the brain. We will now give a somewhat more full description of the eye.

The following two drawings of the eye, being vertical section and horizontal section of the right eye, seen from above, should be carefully studied.

(i) Fig. 70. (ii)



Vertical section. Horizontal section—right eye seen from above.

a The cornea, transparent, continuous with *g* the sclerotic coat, opaque.

b The anterior or aqueous chamber, filled with aqueous fluid, refractive index *less* than in cornea.

c The iris, surrounding the pupil, which is the hole in the iris.

d The crystalline lens.

e The posterior or vitreous chamber, containing the semi-fluid vitreous humour.

f The choroid coat of the inside of the eyeball, black.

g The sclerotic coat on the outside.

hh' The retina, network of nerve fibrils to receive images —vertical section.

pp' The retina, network of nerve fibrils to receive images —horizontal section.

i The visual centre of the retina, where sight is keenest, also called the "yellow spot."

k The "punctum cœcum," also called the "optic disk."

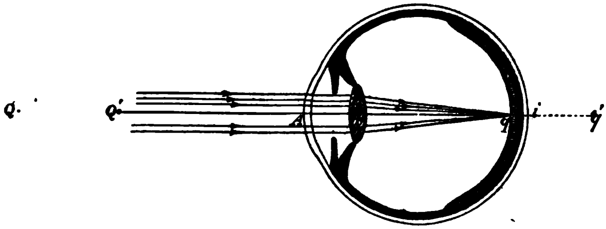
l The optic nerve.

m Continuation of sclerotic, coating the optic nerve.

106. The convex cornea (*a*), receiving, say, parallel rays, causes them to converge: this convergence is slightly diminished at the surface of the aqueous humour (*b*), since its refractive index is less than that of cornea, but again greatly increased at the first surface of the double convex crystalline lens (*d*), and yet again greatly increased at the second surface of the lens, which is considerably denser than the humours before and behind it.

On the whole then we may regard cornea, lens, and humours as a compound object glass, which makes parallel or slightly divergent rays converge to a focus on the retina *hh′*, if the retina is at the proper distance. Now when the eye is perfectly at rest, all its muscles relaxed and no effort at all exerted, the retina *is* at the proper distance to receive the image formed by parallel rays incident on the cornea, is in fact in the principal focus of the compound object glass above mentioned.

Fig. 71.



Thus, suppose the eye is directed to a point Q a long way off, so that Q is on the *optical axis* of the eye, i.e. the line *Adi*, where A and d are the centres of the cornea and crystalline lens respectively, and i is the point of keenest sight. The rays proceed as described, and converge to i, and thus excite on the retina a sensation which is conveyed by the nerve fibrils of the retina to the brain.

107. We have, as we said, here supposed the rays to come parallel, i.e. from some very distant object, as the moon. But suppose now that the eye is directed to some object a great deal closer, suppose, for instance, we wish to pick a hair from the end of a pen: and suppose for the moment the eye to remain in exactly the same state as in the figure above.

Then, since an object Q and its image q move in the same direction, if Q is brought to a point Q', q would be brought to a point q' *behind* the retina, and the eye would see no distinct image. This, as we said in Art. (72), constitutes the defect known as "long sight."

If, then, the eye is to see a clear image, i.e. if q' is to fall on the retina, there must evidently be some change in shape, either of the cornea (*a*) or the crystalline lens (*d*), or else the distance between the lens (*d*) and the retina must in some manner or other be altered at will*.

"It has long been disputed what the exact change is, and the dispute is not yet settled. There is a small muscle, called the ciliary muscle, round the base of the membrane (*o*), which attaches the lens to the walls of the eyeball, and modern writers ascribe to this muscle the function of altering the shape and position of the lens, but even so the *modus operandi* remains obscure. A change in the position of the lens implies great compression of the humour in one or other of the chambers of the eye, and it must be no slight force that shall effect this compression.

I believe it is certain, from direct observation, that the lens is pushed forward in near vision, and also that its anterior surface is made more convex. This latter point is determined by watching the image of a lamp flame reflected at the first surface of the lens: the image grows smaller when the eye is adapted for near vision. The small ciliary muscle seems unequal to these effects. More probably they are due to the constriction of the eyeball by two strong muscles called the "oblique," which gird the ball in a plane at right angles to its axis. The contraction of this girth gives a slight elongation to the eyeball in the direction of its axis, i. e. increases the distance between the lens and the retina, and therefore fits the latter to receive the receding image of a nearer object. You can feel in your own eyes that in withdrawing their gaze from a distant object to an object very near you are performing a muscular exercise; and in straining the eyes to the limits of near vision the muscular effort becomes very great, and may even be made decidedly painful; whereas in re-adapting the eyes to distant vision you feel that you are relaxing a muscle; you cannot strain your eyes for distant vision; the effort is only the effort of abstaining from muscular action, and giving the eyeball time to resume its shape. The limit of distinct near vision is about 6 inches, differing in different people. Some eyes have an undue convexity of cornea or lens, or an undue distance between lens and retina, so that parallel rays incident on the cornea are brought to a focus before they reach the retina. This defect is called "short sight," and cannot be remedied by any effort of the eye, though it is palliated by a contraction (instinctive, involuntary) of the pupil, or by almost complete closure of the eyelids, by which the marginal rays of the pencil are excluded, and the marginal rays are the ones that make confusion and indistinctness in the image. This accounts for the fact of a short-sighted person habitually compressing his eyelids till there is a mere chink between them. But the malformation is easily remedied by the interposition of a double-concave or plano-concave eye-glass in front of the eye, which gives a slight divergence to the parallel rays from a distant object and throws the image of that object further back into the eye till it is within the range of the retina."

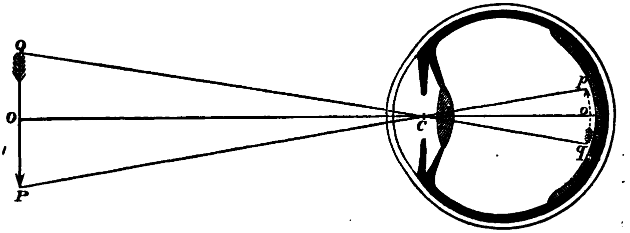
---

* Both of these changes would clearly aid in bringing q' to q.

"Long-sight," or inability to see clearly objects that are within a foot or two feet, is due to loss of muscular power in the adjustment of the eye for near vision, and is a very common defect in old people. Also in advancing age there is a flattening of the cornea, which throws images a little further back, and makes a greater effort required for bringing the retina into the focus. This is remedied by a convex eye-glass, such as old people generally use.

108. We give a figure shewing the formation of the image of an object of any size; only one point of which can of course lie upon the optical axis. The result is precisely as in a double convex lens, the centre of the eye, that is, the optical centre, lying at about the middle point of the pupil *b*.

Fig. 72.



Thus let *POQ* be an object and let *C* be the optical centre of the compound lens. Then

$$\frac{1}{Cp} - \frac{1}{CP} = \frac{1}{f}; \quad \frac{1}{Cq} - \frac{1}{CQ} = \frac{1}{f}, \text{ \&c.}$$

Range of Vision.

109. When the eye looks straight forward, full face, its range of vision is much limited; above, by the projecting lid and eyebrow; below, by the projecting lid and cheekbone; inwards, by the projecting nose: but outwards there is no hindrance at all; the most oblique rays from the right meet with no obstacle to prevent their entrance into the right eye; and in like manner the left eye is open to the most oblique rays from the left. Exactly corresponding to this ordinary range of admission of rays from outer objects, is the range of sensitiveness possessed by the retina. This is shewn in figures (i) and (ii). The vertical section shews the range of retinal sensitiveness above and below, and the horizontal section shews the range inward and outward. In the latter it will be noticed that the portion *ih'* of the retina of the right eye is much greater than the portion *ih*, *i* being the visual centre of the retina, in the axis of vision. *ih'* receives rays from the right, unhindered: *ih* receives rays from the left, but the nose being in the way, it does not receive those of great obliquity, and there is therefore no need of retina for such rays. As nearly as I can estimate, the upward range is about 60°, the downward about 50°, the inward about 50°, the outward about 95°. Looking straight forwards, one can detect objects to right or left further back than the horizontal line joining the two eyes.

110. Over the entrance of the optic nerve (Fig. 72 (ii)), there is a circular patch of the retina which is incapable of distinct vision, though not insensible to general impressions of light and colour. Looking with only one eye open, at very large print, it may be noticed that there is an obscuration of a batch of letters about 15° to the outer side of the centre of vision (to the left, looking with the left eye; to the

right, looking with the right eye). This patch of obscurity is about 5° in diameter. Looking with both eyes open, the blindness of each punctum cœcum is corrected by the sharp-sighted tract on the corresponding side of the other retina.

It is pretty well established that the left halves of the two retinæ are supplied with nerve fibres from the left half of the brain, and the right halves from the right. The evidence of this is got partly from dissection, and partly from observation of morbid phenomena: instances are recorded in which disease on one side of the brain was attended with permanent blindness of the corresponding sides of both retinæ. "Half-blindness" of a transient nature, affecting both eyes in the same way, points to the same conclusion. [Read an interesting paper by Dr Wollaston in the *Philosophical Transactions* of the Royal Society, 1824, I. p. 222.]

111. It will be observed that images on the retina are upside down, and we might therefore expect to see everything upside down. But since we see *everything* upside down, this effect is destroyed. "What has the mind to do with upside down so long as it gets evidence of position and motion of things external *in harmony* with the uprightness of the body, and the fact of gravity pulling everything to the earth?"

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